Perspective projection

A. Mantegna, *Martyrdom of St. Christopher*, c. 1450
Overview of next two lectures

• The pinhole projection model
  • Qualitative properties
  • Perspective projection matrix

• Cameras with lenses
  • Depth of focus
  • Field of view
  • Lens aberrations

• Digital sensors
Let’s design a camera

Idea 1: put a piece of film in front of an object
Do we get a reasonable image?
Pinhole camera

Add a barrier to block off most of the rays
Pinhole camera

- Captures **pencil of rays** – all rays through a single point: aperture, center of projection, optical center, focal point, camera center
- The image is formed on the **image plane**
Pinhole cameras are everywhere
Pinhole cameras are everywhere

Tree shadow during a solar eclipse

photo credit: Nils van der Burg

http://www.physicstogo.org/index.cfm

Slide by Steve Seitz
Camera obscura

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Source: A. Efros
Turning a room into a camera obscura

After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

After his done inside, it gets harder. “I leave the room and I am constantly checking the weather, I’m hoping the maid reads my note not to come in, I’m worrying that the sun will hit the plastic masking and it will fall down, or that I didn’t trigger the lens.”

From Grand Images Through a Tiny Opening, Photo District News, February 2005

Abelardo Morell, Camera Obscura Image of Manhattan View Looking South in Large Room, 1996

https://www.abelardomorell.net/camera-obscura
Turning a room into a camera obscura

My hotel room, contrast enhanced.  The view from my window

Accidental pinholes produce images that are unnoticed or misinterpreted as shadows

A. Torralba and W. Freeman, Accidental Pinhole and Pinspeck Cameras, CVPR 2012
Modeling projection

- How do we find the projection $P'$ of a scene point $P$?
  - Form the **visual ray** connecting $P$ to the camera center $O$ and find where it intersects the image plane.
- All scene points that lie on this visual ray have the same projection in the image.
- Are there scene points for which this projection is undefined?
Overview

- Pinhole projection model
  - Qualitative properties
  - Perspective projection matrix
- Cameras with lenses
Modeling projection

Canonical coordinate system

- The optical center \( (O) \) is at the origin
- The z axis is the *optical axis* perpendicular to the image plane
- The xy plane is parallel to the image plane, x and y axes are horizontal and vertical directions of the image plane
Deriving projected coordinates

Note: instead of dealing with an image that is upside down, most of the time we will pretend that the image plane is *in front* of the camera center
Deriving projected coordinates

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Properties of projection

- Real-world sizes (lengths) are *not* preserved in projection
  - What other properties are/are not preserved?

Piero della Francesca, *Flagellation of Christ*, 1455-1460
Projection of lines

- What happens to parallel lines in projection?

Piero della Francesca, *Flagellation of Christ*, 1455-1460
Projection of lines

- What happens to parallel lines in projection?

Image source: Piero della Francesca, *Flagellation of Christ*, 1455-1460
Constructing vanishing points

- What about another line going in the same direction?
Converging lines are a powerful perspective cue
Projection of lines

• When is parallelism preserved?

Piero della Francesca, *Flagellation of Christ*, 1455-1460
Projection of lines

• When is parallelism preserved?
  • When the parallel lines are also parallel to the image plane

Piero della Francesca, *Flagellation of Christ*, 1455-1460
Projection of planes

- Patterns on non-fronto-parallel planes are distorted by a transformation called a homography.

Piero della Francesca, *Flagellation of Christ*, 1455-1460
Projection of planes

• What about patterns on *fronto-parallel planes*?

\[(x, y, z) \to \left(f \frac{x}{z}, f \frac{y}{z}\right)\]

• All points on a fronto-parallel plane are at a fixed depth \(z\)
• The pattern gets scaled by a factor of \(f / z\), but angles and ratios of lengths/areas are preserved

Piero della Francesca, *Flagellation of Christ*, 1455-1460
Vanishing lines of planes

- Each family of parallel planes is associated with a *vanishing line* in the image
- How can we construct the vanishing line of a plane?
Vanishing lines of planes

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- How can we construct the vanishing line of a plane?

Figure source: S. Seitz
Vanishing lines of planes

- **Horizon:** vanishing line of the ground plane
  - What can the horizon tell us about the relative height of scene points and the camera?
Comparing heights
Measuring height

What is the height of the camera?

Slide by Steve Seitz
Projection of 3D shapes

Figure 3. (a) A figure taken from Marr (1982).

J. Koenderink. What does the occluding contour tell us about solid shape? Perception 13 (321-330), 1984
Projection of 3D shapes

Figure 3. (a) A figure taken from Marr (1982). The suggestion is that convexities and concavities in the projection of the snake have to do with relative *distances* rather than with local shapes.

J. Koenderink. *What does the occluding contour tell us about solid shape?* Perception 13 (321-330), 1984
Projection of 3D shapes

J. Koenderink. **What does the occluding contour tell us about solid shape?** Perception 13 (321-330), 1984
Projection of 3D shapes

Figure 4. Details from Dürer’s “Samson killing the lion”. (Bartsch #2; the print dates from 1498.)

J. Koenderink. What does the occluding contour tell us about solid shape? Perception 13 (321-330), 1984
Projection of 3D shapes

• What is the shape of the projection of a sphere?
Projection of 3D shapes

• What is the shape of the projection of a sphere?
Projection of 3D shapes

• Are the widths of the projected columns equal?
  • The exterior columns are wider
  • This is not an optical illusion, and is not due to lens flaws
  • Phenomenon pointed out by Leonardo Da Vinci

Source: F. Durand
Overview

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Modeling projection

Projection equation: \((x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z}\right)\)

- Is this a linear transformation?
  - No – division by \(z\) makes it nonlinear

Source: J. Ponce, S. Seitz
Homogeneous coordinates

- To form homogeneous coordinates from normal Euclidean coordinates, append 1 as the last entry:

\[
\begin{align*}
(x, y) & \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \\
(x, y, z) & \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\end{align*}
\]

homogeneous image coordinates

homogeneous scene coordinates

- To convert from homogeneous coordinates, divide by the last entry:

\[
\begin{align*}
\begin{bmatrix} x \\ y \\ w \end{bmatrix} & \Rightarrow (x/w, y/w) \\
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} & \Rightarrow (x/w, y/w, z/w)
\end{align*}
\]

In homogeneous coordinates, all scalar multiples represent the same point!
Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates:

\[
\begin{bmatrix}
  x \\
  y \\
  z \\
  1
\end{bmatrix}
= \begin{bmatrix}
  ? \\
  ? \\
  ? \\
  ?
\end{bmatrix} \Rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)
\]

In practice: lots of coordinate transformations…

\[
\begin{bmatrix}
  2D \\
  \text{point} \\
  (3x1)
\end{bmatrix}
= \begin{bmatrix}
  \text{Camera to pixel coord.} \\
  \text{trans. matrix} \\
  (3x3)
\end{bmatrix}
\begin{bmatrix}
  \text{Perspective} \\
  \text{projection matrix} \\
  (3x4)
\end{bmatrix}
\begin{bmatrix}
  \text{World to} \\
  \text{camera coord.} \\
  \text{trans. matrix} \\
  (4x4)
\end{bmatrix}
\begin{bmatrix}
  3D \\
  \text{point} \\
  (4x1)
\end{bmatrix}
\]
Orthographic Projection

Special case of perspective projection

- Distance from center of projection to image plane is infinite
- Also called “parallel projection”

Slide by Steve Seitz
Orthographic Projection

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Orthographic Projection

Special case of perspective projection

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Assuming projection along the z axis, what’s the matrix?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} =
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]