Optical flow

Many slides adapted from S. Seitz, R. Szeliski, M. Pollefeys
Outline

• Motion: motivation
• Motion field and optical flow definition
• Brightness constancy constraint, aperture problem
• Estimating optical flow (Lucas-Kanade)
• Beyond basic flow estimation
• Applications
Motion is a powerful perceptual cue

- Sometimes, it is the only cue
Motion is a powerful perceptual cue

- Even “impoverished” motion data can evoke a strong percept

Motion is a powerful perceptual cue

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Uses of motion in computer vision

- Video analysis and enhancement (stabilization, shot boundary detection, motion magnification, etc.)
- Object tracking and segmentation in videos
- Structure from motion
- Event and activity recognition
- Self-supervised and predictive learning

Source: Tomasi & Kanade

Source: Walker et al.
Motion field vs. optical flow

- The **motion field** is the projection of the 3D scene motion into the image.
Motion field vs. optical flow

- The **motion field** is the projection of the 3D scene motion into the image.
- **Optical flow** is the apparent motion of brightness patterns in the image.
- Are optical flow and motion field the same?
  - Apparent motion can be caused by lighting changes without any actual motion. E.g., consider a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination.
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Estimating optical flow

• Given frames at times $t - 1$ and $t$, estimate the apparent motion field $u(x, y)$ and $v(x, y)$ between them.

• **Brightness constancy constraint**: projection of the same point looks the same in every frame:

$$I(x, y, t - 1) = I(x + u(x, y), y + v(x, y), t)$$

• Additional assumptions:
  • **Small motion**: points do not move very far
  • **Spatial coherence**: points move like their neighbors
Estimating optical flow

• Brightness constancy constraint:

\[ I(x, y, t - 1) = I(x + u(x, y), y + v(x, y), t) \]

• Linearize the right-hand side using Taylor expansion:

\[ I(x, y, t - 1) \approx I(x, y, t) + I_x u(x, y) + I_y v(x, y) \]
\[ I_x u(x, y) + I_y v(x, y) + I(x, y, t) - I(x, y, t - 1) = 0 \]

What could this be?

• Hence, \( I_x u(x, y) + I_y v(x, y) + I_t = 0 \)
The brightness constancy constraint

\[ I_x u(x, y) + I_y v(x, y) + I_t = 0 \]

- Given the gradients \( I_x, I_y \) and \( I_t \), can we uniquely recover the motion \((u, v)\)?
  - Suppose \((u, v)\) satisfies the constraint: \( \nabla I \cdot (u, v) + I_t = 0 \)
  - Then \( \nabla I \cdot (u + u', v + v') + I_t = 0 \) for any \((u', v')\) s.t. \( \nabla I \cdot (u', v') = 0 \)
  - Interpretation: the component of the flow perpendicular to the gradient (i.e., parallel to the edge) cannot be recovered!
The aperture problem

Perceived motion
The aperture problem

Actual motion
The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion
The barber pole illusion

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Solving the aperture problem

\[ I_x u(x, y) + I_y v(x, y) + I_t = 0 \]

- How to get more equations for a pixel?
  - **Spatial coherence constraint:** assume the pixel’s neighbors have the same \((u, v)\)
  - If we have \(n\) pixels in the neighborhood, then we can set up a linear least squares system:

\[
\begin{bmatrix}
I_x(x_1, y_1) & I_y(x_1, y_1) \\
\vdots & \vdots \\
I_x(x_n, y_n) & I_y(x_n, y_n)
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
v \\
v
\end{bmatrix}
= -
\begin{bmatrix}
I_t(x_1, y_1) \\
\vdots \\
I_t(x_n, y_n)
\end{bmatrix}
\]
Estimating optical flow

\[
\begin{bmatrix}
I_x(x_1, y_1) & I_y(x_1, y_1) \\
\vdots & \vdots \\
I_x(x_n, y_n) & I_y(x_n, y_n)
\end{bmatrix}
\begin{bmatrix}
u \\
\end{bmatrix}
= -
\begin{bmatrix}
I_t(x_1, y_1) \\
\vdots \\
I_t(x_n, y_n)
\end{bmatrix}
\]

\[
A
\quad b
\]

• Solution:

\[
\begin{bmatrix}
\Sigma I_x I_x & \Sigma I_x I_y \\
\Sigma I_x I_y & \Sigma I_y I_y
\end{bmatrix}
\begin{bmatrix}
u \\
\end{bmatrix}
= -
\begin{bmatrix}
\Sigma I_x I_t \\
\Sigma I_y I_t
\end{bmatrix}
\]

\[
A^T A
\quad A^T b
\]

What does \(A^T A\) remind you of?

• When is the system solvable?
Recall: second moment matrix

- Estimation of optical flow is well-conditioned precisely for regions with high “cornerness”:

\[
\lambda_1 \text{ and } \lambda_2 \text{ are large, } \lambda_1 \sim \lambda_2
\]

\[
\lambda_1 \text{ and } \lambda_2 \text{ are small}
\]

\[
\lambda_1 \gg \lambda_2
\]

\[
\lambda_2 \gg \lambda_1
\]
Conditions for solvability

- “Bad” case: single straight edge
Conditions for solvability

- “Good” case
Lucas-Kanade flow

\[
\begin{bmatrix}
I_x(x_1, y_1) & I_y(x_1, y_1) \\
\vdots & \vdots \\
I_x(x_n, y_n) & I_y(x_n, y_n)
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -\begin{bmatrix}
I_t(x_1, y_1) \\
\vdots \\
I_t(x_n, y_n)
\end{bmatrix}
\]

- Solution is given by \((A^T A)^{-1}A^T b\)
- Lucas-Kanade flow:
  - Find \((u, v)\) minimizing \(\sum_i (I_t(x_i + u, y_i + v) - I_{t-1}(x_i, y_i))^2\),
    use Taylor approximation of \(I_t(x_i + u, y_i + v)\) for small shifts \((u, v)\)
    to obtain closed-form solution

B. Lucas and T. Kanade. *An iterative image registration technique with an application to stereo vision.*
Lucas-Kanade flow example

Source: MATLAB Central File Exchange
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When does Lucas-Kanade fail?

- The motion is large (larger than a pixel)
- A point does not move like its neighbors
- Brightness constancy does not hold
Fixing the errors in Lucas-Kanade

- The motion is large
Fixing the errors in Lucas-Kanade

• The motion is large
  • Multi-resolution estimation, iterative refinement
Fixing the errors in Lucas-Kanade

- The motion is large
  - Multi-resolution estimation, iterative refinement
Fixing the errors in Lucas-Kanade

- The motion is large (larger than a pixel)
- A point does not move like its neighbors
  - Motion segmentation

Figure 11: (a) The optic flow from multi-scale gradient method. (b) Segmentation obtained by clustering optic flow into affine motion regions. (c) Segmentation from consistency checking by image warping. Representing moving images with layers.

Fixing the errors in Lucas-Kanade

- The motion is large (larger than a pixel)
- A point does not move like its neighbors
- Brightness constancy does not hold
  - Feature matching (e.g., SIFT) or tracking
Shi-Tomasi feature tracker

- Find good features using eigenvalues of second-moment matrix
  - Key idea: “good” features to track are the ones whose motion can be estimated reliably
- From frame to frame, track with Lucas-Kanade
  - This amounts to assuming a translation model for frame-to-frame feature movement
- Check consistency of tracks by affine registration to the first observed instance of the feature
  - Affine model is more accurate for larger displacements
  - Comparing to the first frame helps to minimize drift

Tracking example

Figure 1: Three frame details from Woody Allen’s *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.

Figure 2: The traffic sign windows from frames 1, 6, 11, 16, 21 as tracked (top), and warped by the computed deformation matrices (bottom).

State-of-the-art optical flow estimation

- Current best methods are learned (often on synthetic data)

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Synthetic dataset: Flying Chairs

State-of-the-art optical flow estimation

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Z. Teed and J. Deng. **RAFT: Recurrent All-Pairs Field Transforms for Optical Flow**. ECCV 2020 (Best Paper Award)
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Motion magnification

Idea: take flow, magnify it

Source: D. Fouhey and J. Johnson

C. Liu et al., Motion Magnification, SIGGRAPH 2005
Motion magnification

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Motion magnification

(a) Registered input frame  (b) Clustered trajectories of tracked features  (c) Layers of related motion and appearance

(d) Motion magnified, showing holes  (e) After texture in-painting to fill holes  (f) After user's modification to segmentation map in (c)

Source: D. Fouhey and J. Johnson

C. Liu et al., Motion Magnification, SIGGRAPH 2005
Motion magnification

(a) Input

(b) Magnified

H. Wu et al. Eulerian Video Magnification for Revealing Subtle Changes in the World, SIGGRAPH 2012
Activity recognition

- Optical Flow is sometimes used as an input feature for video classification with CNNs


Source: D. Fouhey and J. Johnson