

Corner detection



9300 Harris Corners Pkwy, Charlotte, NC

Outline

- Keypoint detection: Motivation
- Deriving a corner detection criterion
- The Harris corner detector
- Invariance properties of corners

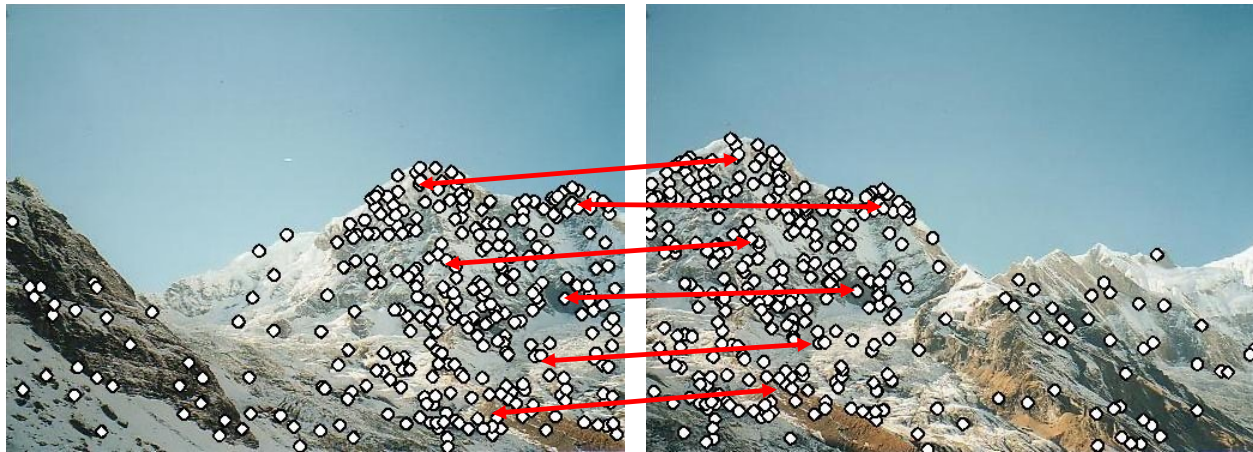
Why extract keypoints?

- Motivation: image alignment
 - We have two images – how do we combine them?



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Step 1: extract keypoints

Step 2: match keypoint features

Why extract keypoints?

- Motivation: image alignment
 - We have two images – how do we combine them?



Step 1: extract keypoints

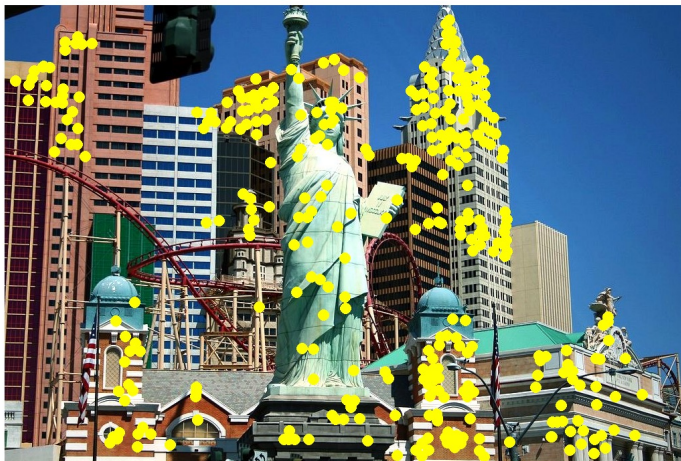
Step 2: match keypoint features

Step 3: align images

Applications

Keypoints are useful for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Database indexing and retrieval

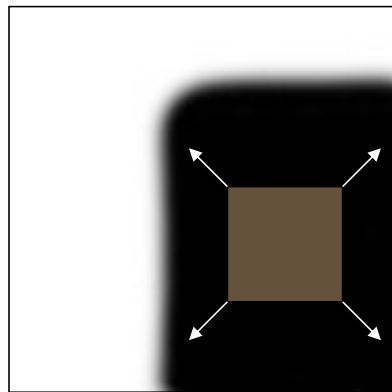


Corner detection: Outline

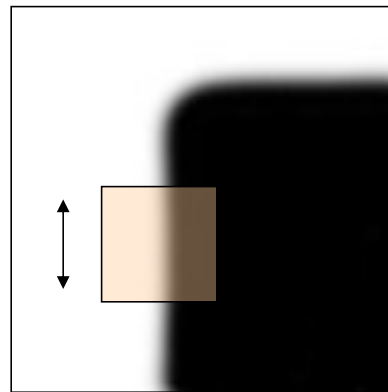
- Motivation
- Deriving a corner detection criterion

Deriving a corner detection criterion

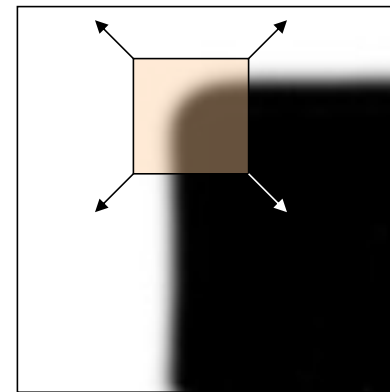
- Basic idea: we should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give a *large change* in intensity



“flat” region:
no change in
all directions



“edge”:
no change
along the edge
direction

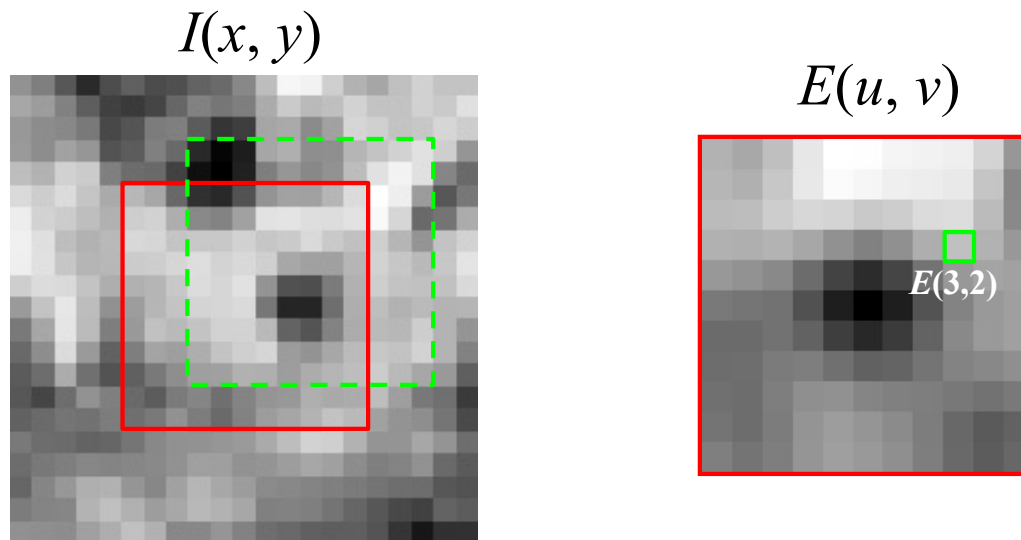


“corner”:
significant
change in all
directions

Deriving a corner detection criterion

- Change in appearance of window W for the shift (u, v) :

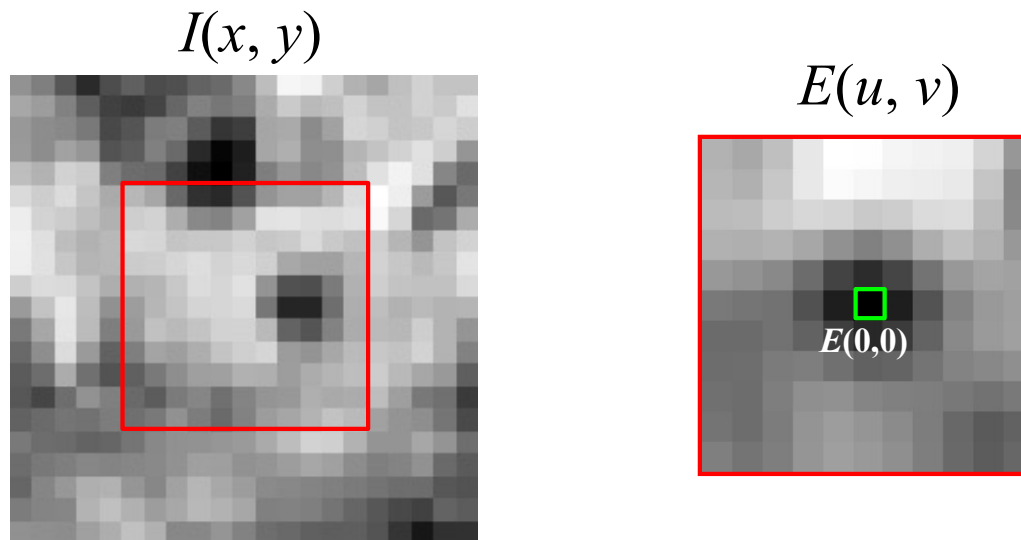
$$E(u, v) = \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2$$



Deriving a corner detection criterion

- Change in appearance of window W for the shift (u, v) :

$$E(u, v) = \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2$$



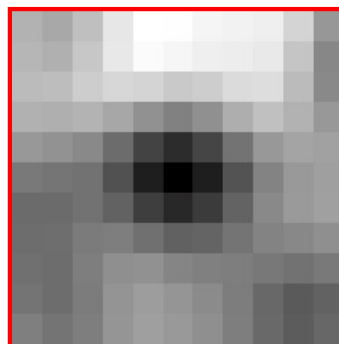
Deriving a corner detection criterion

- Change in appearance of window W for the shift (u, v) :

$$E(u, v) = \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

- We want to find out how this function behaves for small shifts

$E(u, v)$

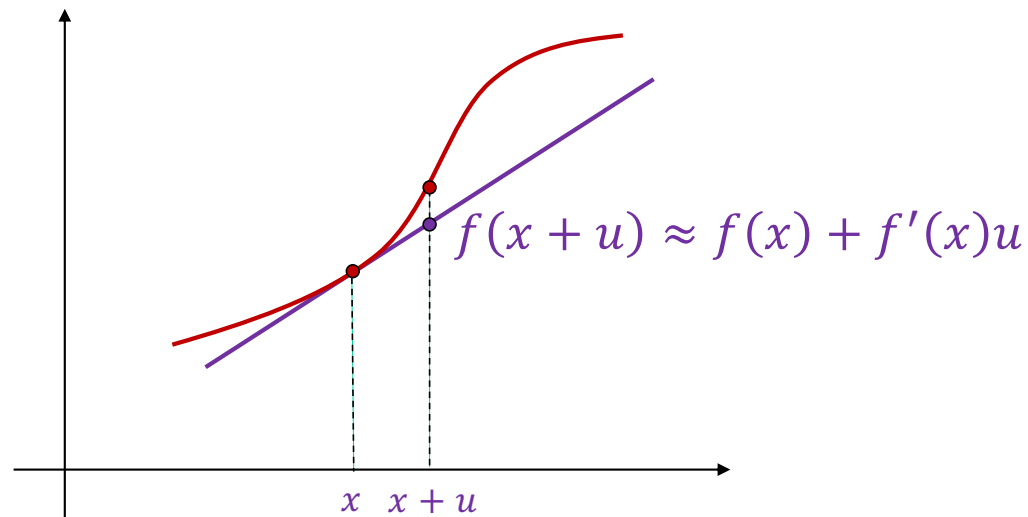


Deriving a corner detection criterion

- First-order Taylor approximation for small shifts (u, v) :

$$I(x + u, y + v) \approx I(x, y) + I_x u + I_y v$$

- Recall: first-order Taylor approximation for a 1D function:



Deriving a corner detection criterion

- First-order Taylor approximation for small shifts (u, v) :

$$I(x + u, y + v) \approx I(x, y) + I_x u + I_y v$$

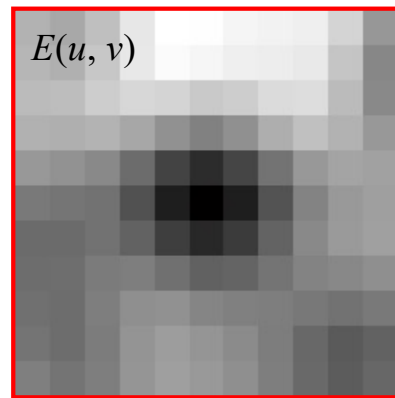
- Let's plug this into $E(u, v)$:

$$\begin{aligned} E(u, v) &= \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2 \\ &\approx \sum_{(x,y) \in W} [I(x, y) + I_x u + I_y v - I(x, y)]^2 \\ &= \sum_{(x,y) \in W} [I_x u + I_y v]^2 = \sum_{(x,y) \in W} I_x^2 u^2 + 2I_x I_y uv + I_y^2 v^2 \end{aligned}$$

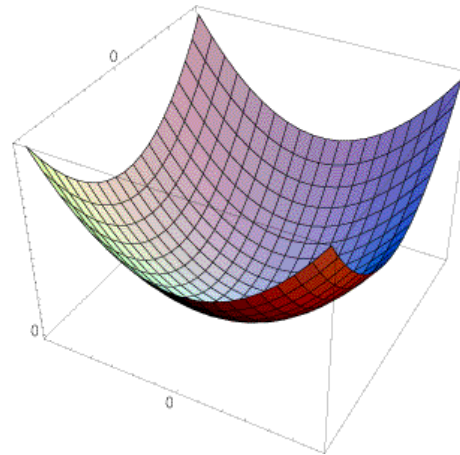
Deriving a corner detection criterion

$$E(u, v) \approx u^2 \sum_{x,y} I_x^2 + 2uv \sum_{x,y} I_x I_y + v^2 \sum_{x,y} I_y^2$$

- This is a quadratic function of (u, v) :



≈



- How can we analyze the shape of this surface?

Deriving a corner detection criterion

$$E(u, v) \approx u^2 \sum_{x,y} I_x^2 + 2uv \sum_{x,y} I_x I_y + v^2 \sum_{x,y} I_y^2$$

$$= (u \quad v) \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

*Second moment
matrix M*

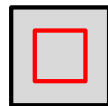
Deriving a corner detection criterion

- Let's consider the simple case of a diagonal M :

$$E(u, v) \approx (u \quad v) \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = (u \quad v) \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

- When does this happen?

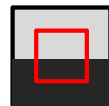
$a = 0, b = 0$



$b = 0$



$a = 0$



$a > 0, b > 0$



Deriving a corner detection criterion

- Let's consider the simple case of a diagonal M :

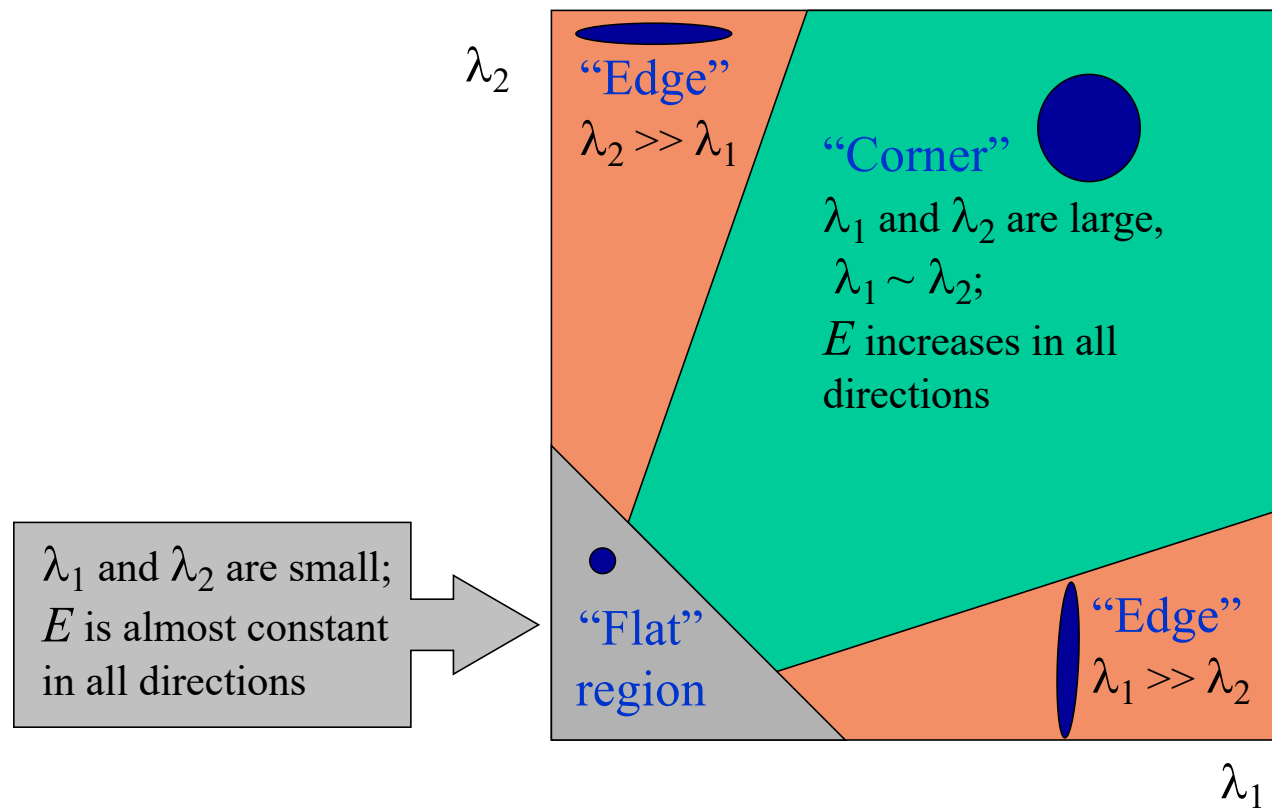
$$E(u, v) \approx (u \quad v) \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

- In general, need to *diagonalize* M and check its eigenvalues:

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

Interpreting the eigenvalues

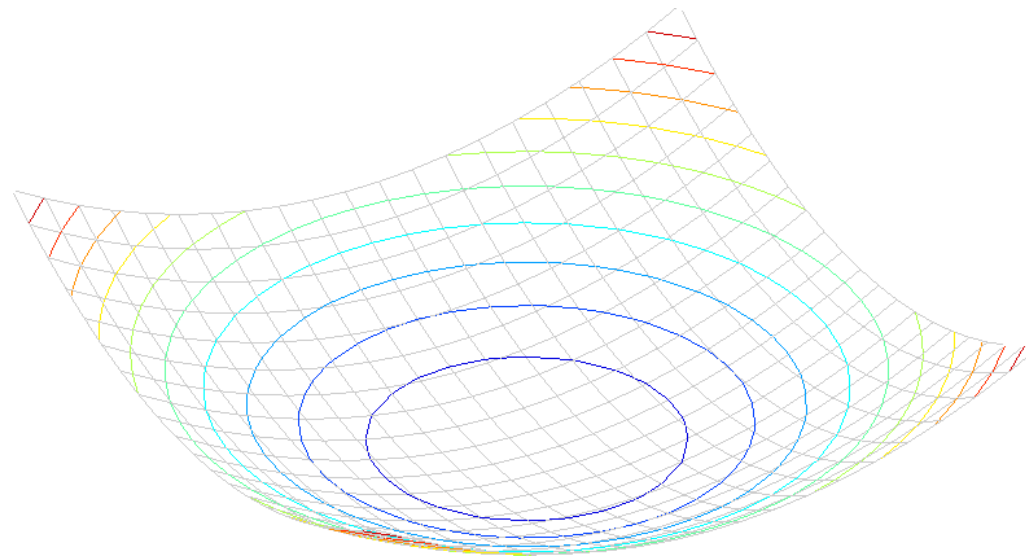
- Classification of image points using eigenvalues of M :



Second moment ellipse

- A “level set” of $E(u, v)$ gives us the equation of an ellipse (in general):

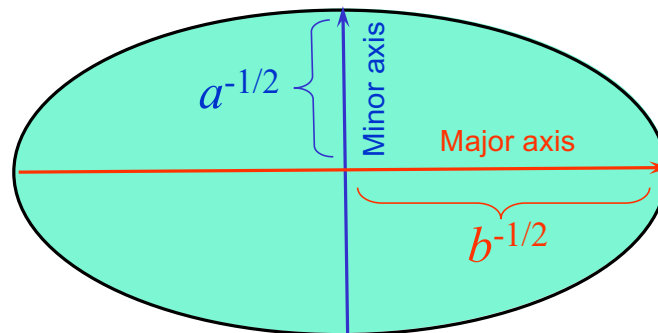
$$(u \quad v)M \begin{pmatrix} u \\ v \end{pmatrix} = \text{const.}$$



Second moment ellipse

- Axis-aligned case:

$$(u \ v) \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = 1$$
$$au^2 + bv^2 = 1$$
$$\frac{u^2}{(a^{-1/2})^2} + \frac{v^2}{(b^{-1/2})^2} = 1$$

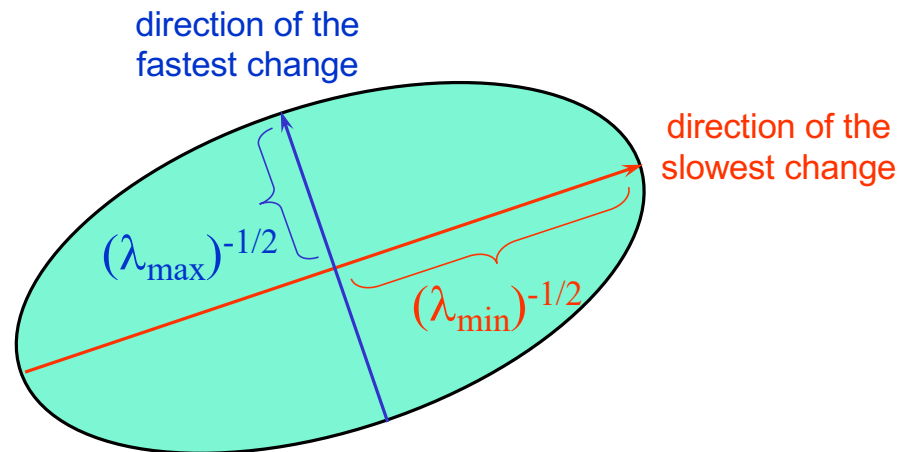


Second moment ellipse

- General case:

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

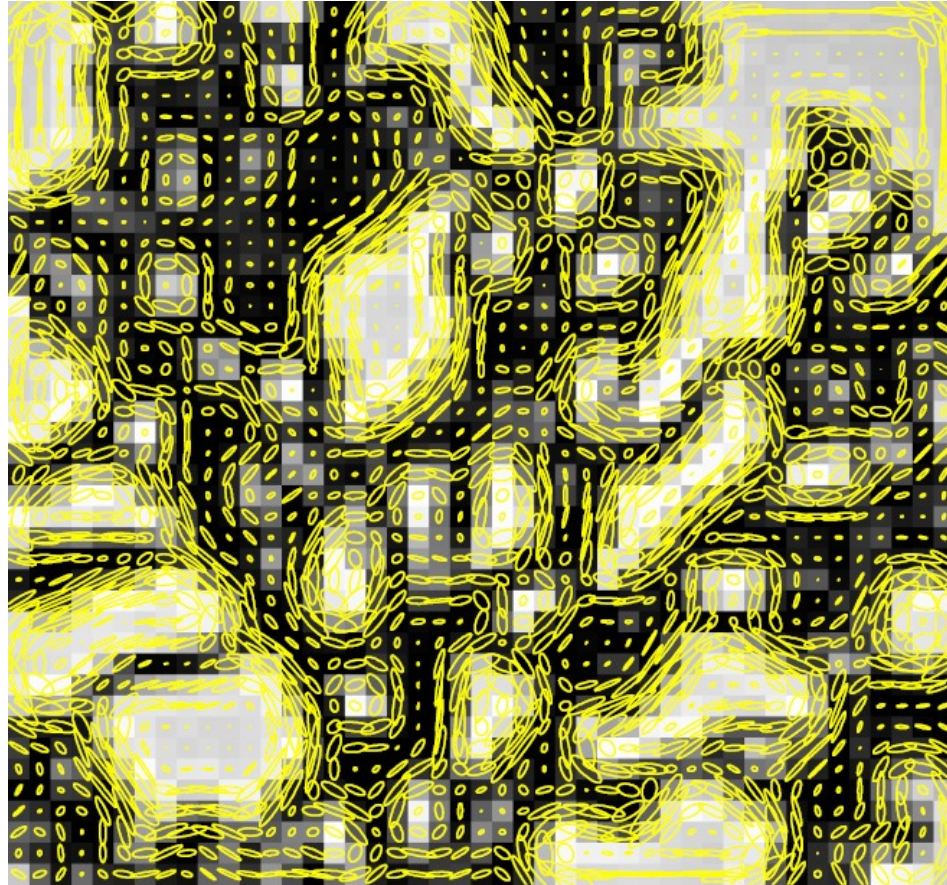
- The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by R



Visualization of second moment ellipses



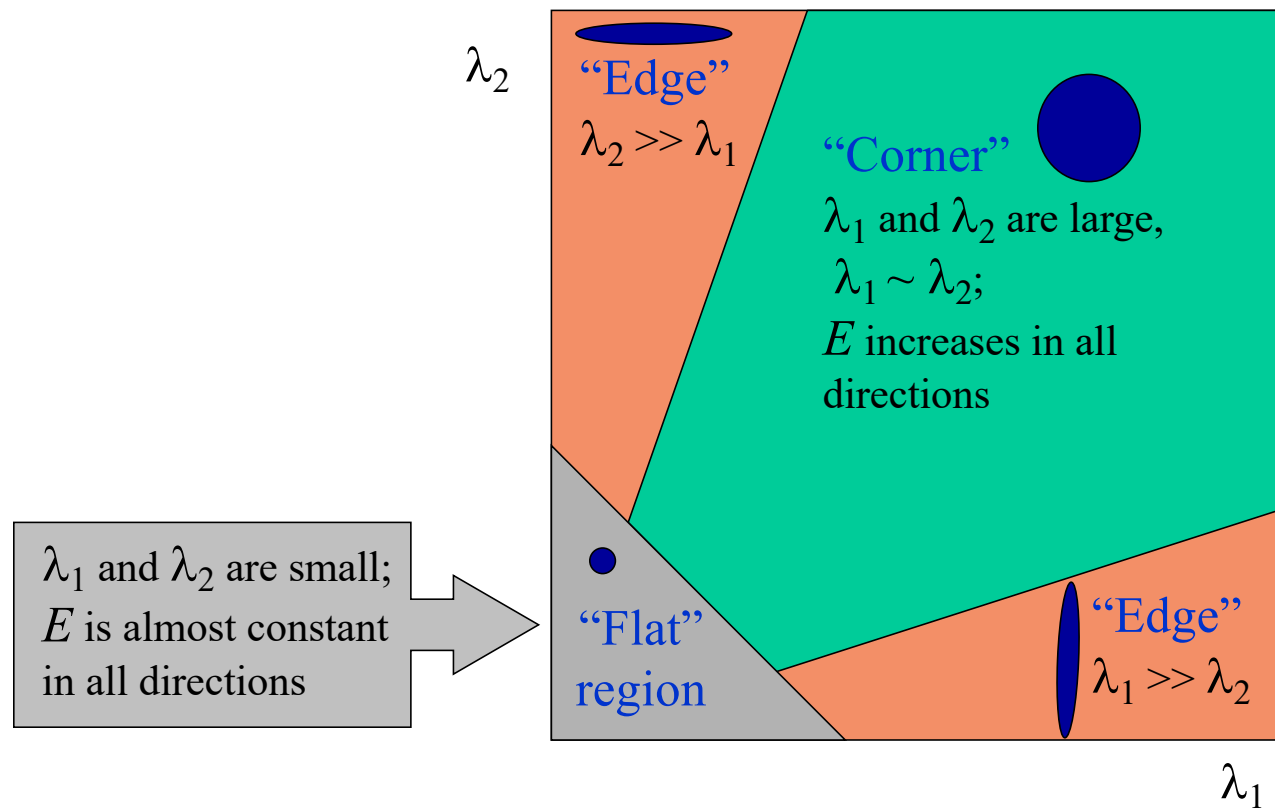
Visualization of second moment ellipses



Note: axes are rescaled so ellipse areas are proportional to edge energy (i.e., bigger ellipses correspond to stronger edges)

Interpreting the eigenvalues

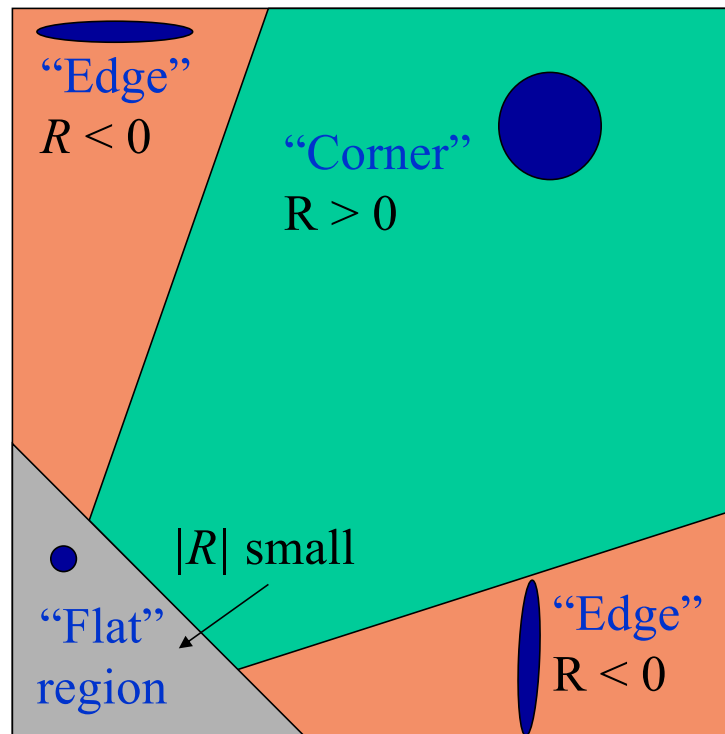
- Classification of image points using eigenvalues of M :



Corner response function

$$R = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α : constant (0.04 to 0.06)



Outline

- Keypoint detection: Motivation
- Deriving a corner detection criterion
- **The Harris corner detector**

The Harris corner detector

1. Compute partial derivatives I_x and I_y at each pixel
2. Compute second moment matrix in a *Gaussian window* around each pixel:

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens, [A Combined Corner and Edge Detector](#),
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

The Harris corner detector

1. Compute partial derivatives I_x and I_y at each pixel
2. Compute second moment matrix in a *Gaussian window* around each pixel
3. Compute corner response function $R = \det(M) - \alpha \text{trace}(M)^2$
4. Threshold R
5. Find local maxima of response function (NMS)

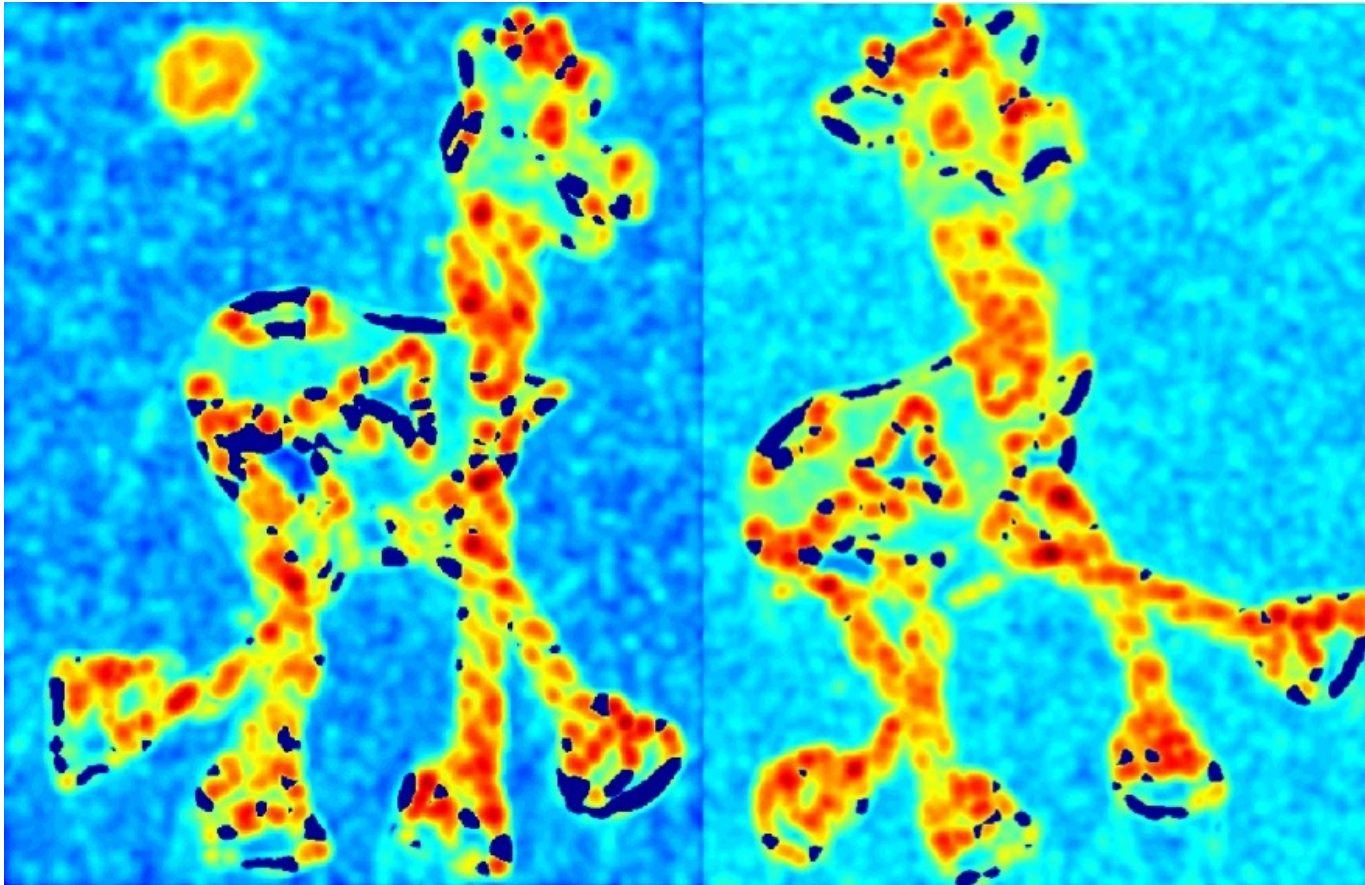
C.Harris and M.Stephens, [A Combined Corner and Edge Detector](#),
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

Harris Detector: Example



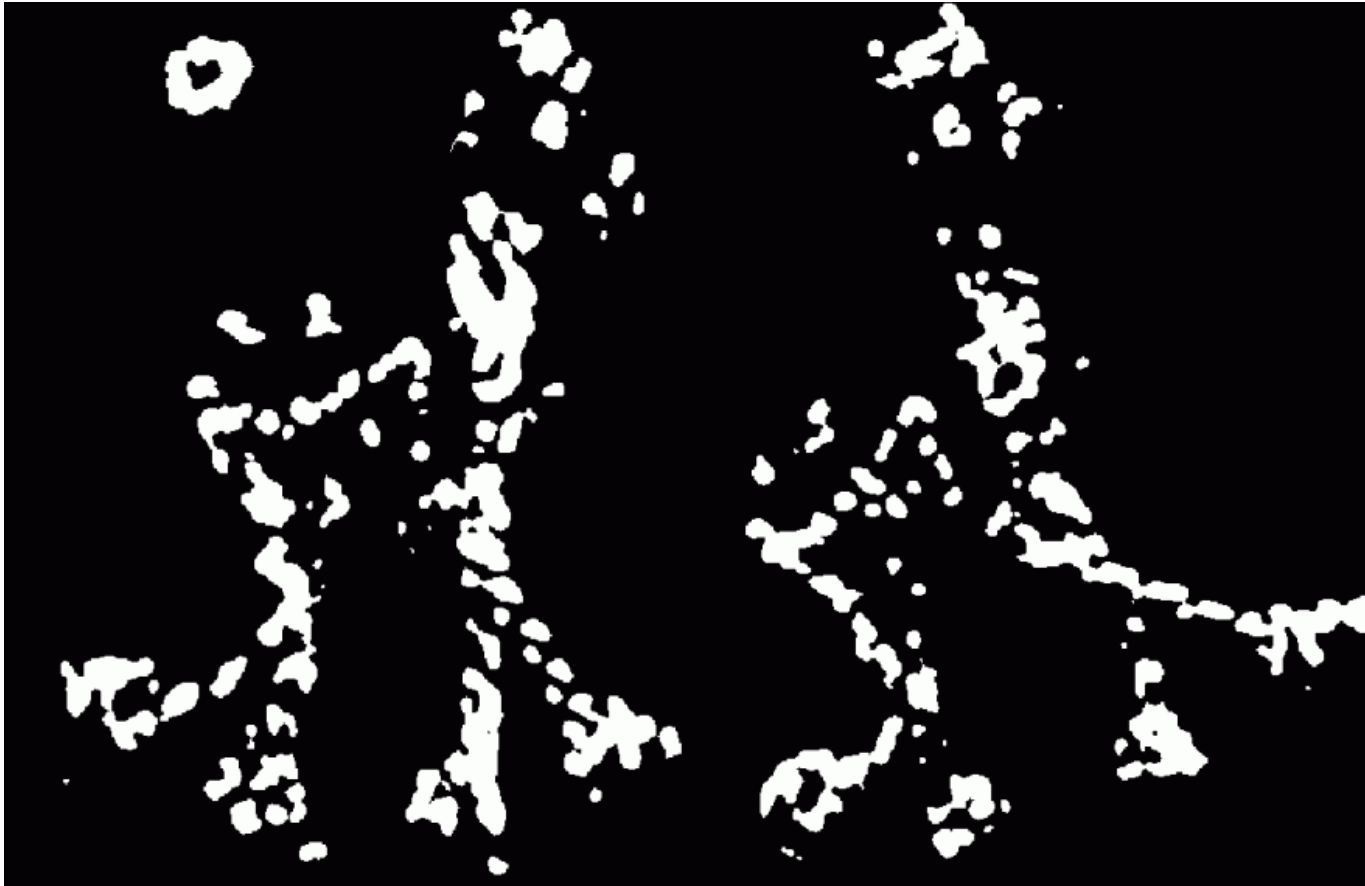
Harris Detector: Example

Compute corner response R



Harris Detector: Example

Find points with large corner response: $R > \text{threshold}$

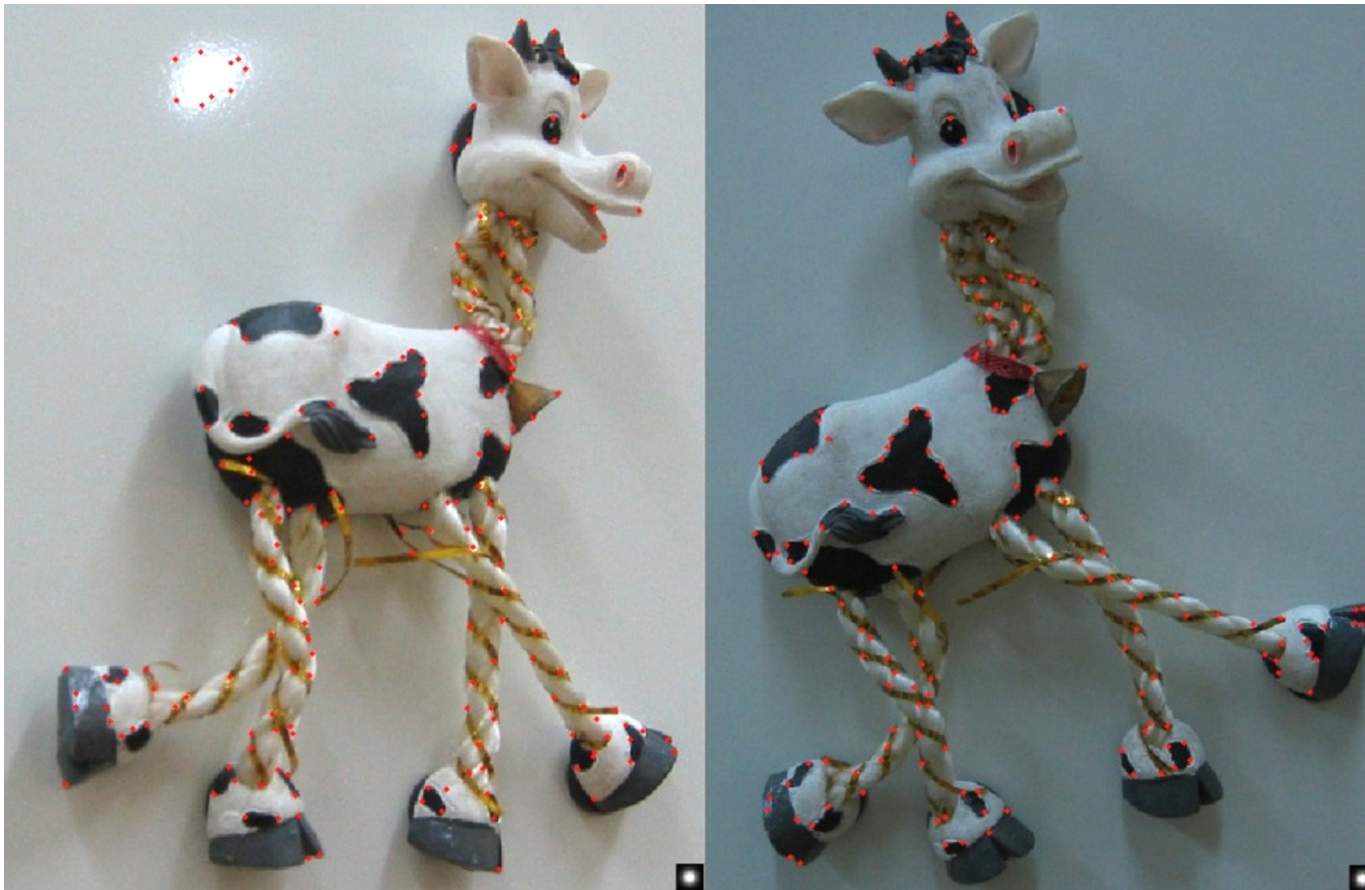


Harris Detector: Example

Take only the points of local maxima of R



Harris Detector: Example



Outline

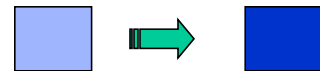
- Keypoint detection: Motivation
- Deriving a corner detection criterion
- The Harris corner detector
- Behavior of corner features w.r.t. image transformations

Behavior w.r.t. image transformations

- To be useful for image matching, “the same” corner features need to show up despite geometric and photometric transformations
- We need to analyze how *the corner response function* and *the corner locations* change in response to various transformations



Affine intensity change



$$I \rightarrow aI + b$$

- What happens to the corner response function in case of intensity shifts ($I \rightarrow I + b$)?
 - It depends only on image derivatives, so it's *invariant* to intensity shifts
- What about intensity scaling ($I \rightarrow aI$)?
 - *Not fully invariant* if threshold stays constant

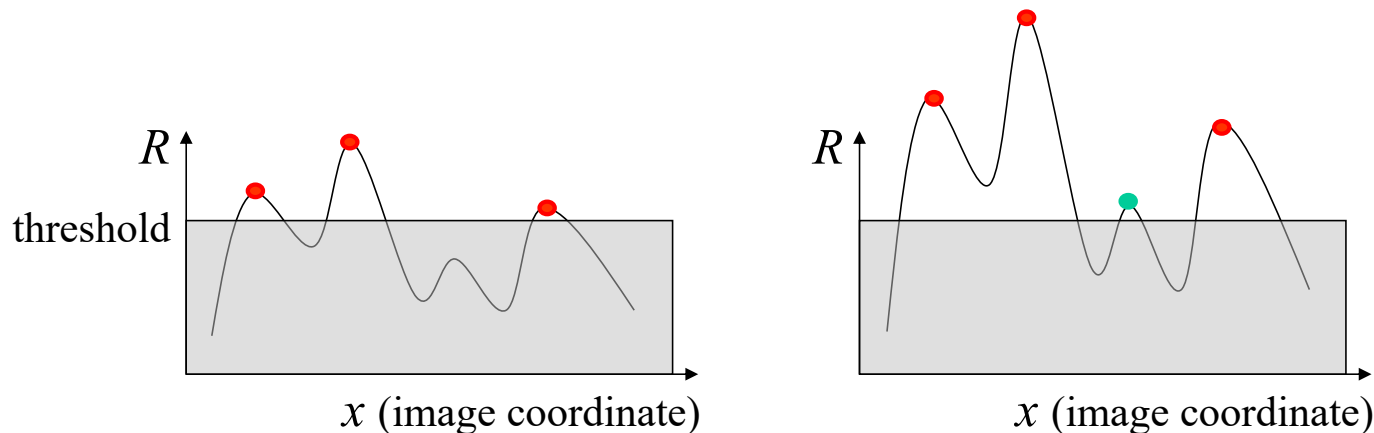
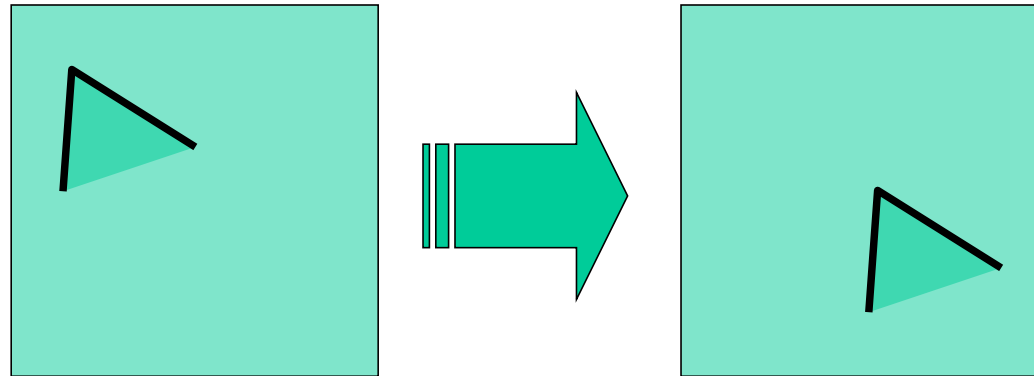
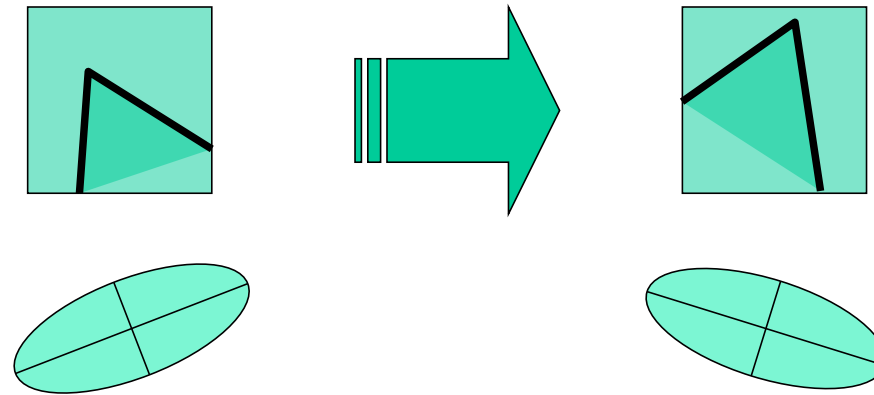


Image translation



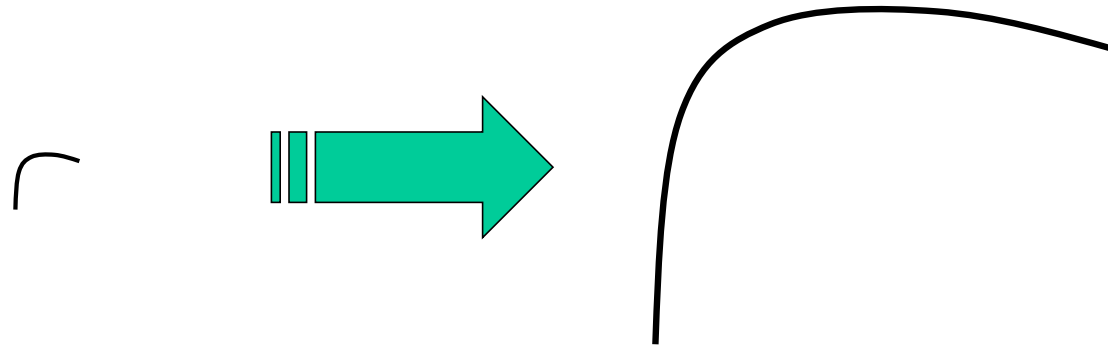
- How do the detected corner locations change if the image pattern is translated?
 - All the ingredients of the second moment matrix are *shift-invariant*, and so is the corner response function
 - However, the locations of the corners are *equivariant (or covariant)* w.r.t. shifts

Image rotation



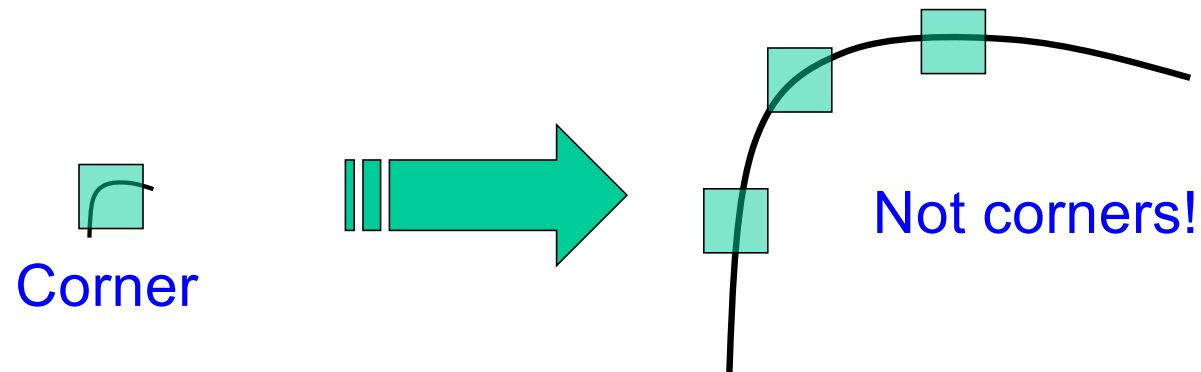
- How do the detected corner locations change if the image pattern is *rotated*?
 - Assuming the second moment matrix is calculated over a circular neighborhood (and ignoring resampling issues), the rotation changes but the eigenvalues stay the same, so the response function is *invariant*
 - The locations of the corners are *equivariant (or covariant)* w.r.t. rotations

Image scaling



- How do the detected corner locations change if the image pattern is *scaled*?

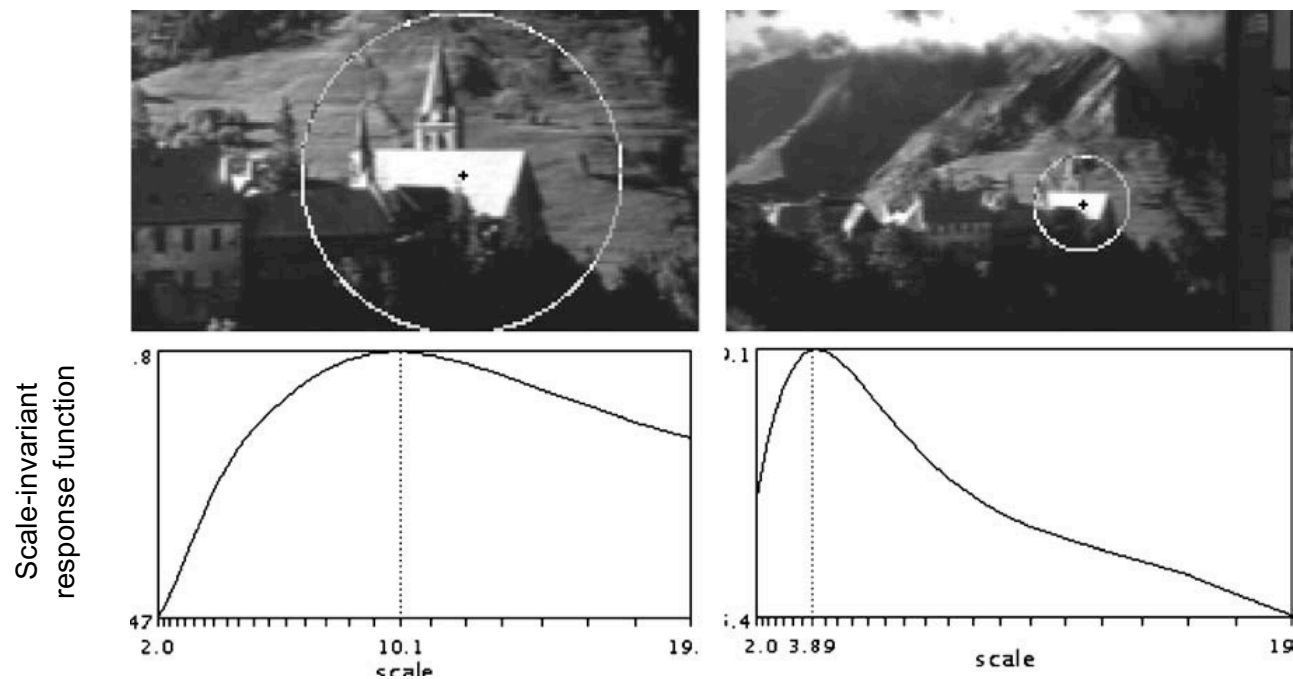
Image scaling



- How do the detected corner locations change if the image pattern is *scaled*?
 - Assuming fixed-size neighborhoods for calculating the second moment matrix, the corner response function is *not invariant* and the corner locations are *not equivariant* w.r.t. scaling

Next: Keypoint detection with scale selection

- We want to extract keypoints with *characteristic scales* that are *equivariant* (or *covariant*) w.r.t. to scaling of the image



K. Mikolajczyk and C. Schmid. [Indexing based on scale invariant interest points](#). ICCV 2001
T. Lindeberg, [Feature detection with automatic scale selection](#), *IJCV* 30(2), pp. 77-116, 1998