Keypoint extraction: Corners

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Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?
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Step 1: extract keypoints
Step 2: match keypoint features
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Step 1: extract keypoints
Step 2: match keypoint features
Step 3: align images
Characteristics of good keypoints

- **Compactness and efficiency**
  - Many fewer keypoints than image pixels

- **Saliency**
  - Each keypoint is distinctive

- **Locality**
  - A keypoint occupies a relatively small area of the image; robust to clutter and occlusion

- **Repeatability**
  - The same keypoint can be found in several images despite geometric and photometric transformations
Applications

Keypoints are used for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Database indexing and retrieval
- Object recognition
Corner Detection: Basic Idea

- We should easily recognize the point by looking through a small window
- Shifting a window in any direction should give a large change in intensity

“flat” region: no change in all directions

“edge”: no change along the edge direction

“corner”: significant change in all directions
Corner Detection: Derivation

Change in appearance of window $W$ for the shift $[u,v]$: 

$$E(u,v) = \sum_{(x,y) \in W} [I(x+u, y+v) - I(x, y)]^2$$
Corner Detection: Derivation

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We want to find out how this function behaves for small shifts

$E(u, v)$
Corner Detection: Derivation

First-order Taylor approximation for small motions \([u, v]\):

\[
I(x + u, y + v) \approx I(x, y) + I_x u + I_y v
\]

Let’s plug this into \(E(u, v)\):

\[
E(u, v) = \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2
\]
Corner Detection: Derivation

$E(u,v)$ can be locally approximated by a quadratic surface:

$$E(u,v) \approx u^2 \sum_{x,y} I_x^2 + 2uv \sum_{x,y} I_x I_y + v^2 \sum_{x,y} I_y^2$$

In which directions does this surface have the fastest/slowest change?
Corner Detection: Derivation

$E(u,v)$ can be locally approximated by a quadratic surface:

$$E(u,v) \approx u^2 \sum_{x,y} I_x^2 + 2uv \sum_{x,y} I_x I_y + v^2 \sum_{x,y} I_y^2$$

$$= \begin{bmatrix} u & v \\ \sum_{x,y} I_x & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Second moment matrix $M$
Interpreting the second moment matrix

A horizontal “slice” of $E(u, v)$ is given by the equation of an ellipse:

$$\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$
Consider the axis-aligned case (gradients are either horizontal or vertical):

\[
M = \begin{bmatrix}
\sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\
\sum_{x,y} I_x I_y & \sum_{x,y} I_y^2
\end{bmatrix}
\]

\[
\begin{bmatrix}
u \\
v
\end{bmatrix}
\begin{bmatrix}
a & 0 \\
0 & b
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix} = 1
\]
Interpreting the second moment matrix

Consider the axis-aligned case (gradients are either horizontal or vertical):

\[
M = \begin{bmatrix}
\sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\
\sum_{x,y} I_x I_y & \sum_{x,y} I_y^2
\end{bmatrix} = \begin{bmatrix}
a & 0 \\
0 & b
\end{bmatrix}
\]

If either \( a \) or \( b \) is close to 0, then this is \textbf{not} a corner, so we want locations where both are large.
Interpreting the second moment matrix

In the general case, need to *diagonalize* $M$:

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by $R$:
Visualization of second moment matrices
Visualization of second moment matrices
Interpreting the eigenvalues

Classification of image points using eigenvalues of $M$:

- $\lambda_1$ and $\lambda_2$ are large, $\lambda_1 \sim \lambda_2$; $E$ increases in all directions
- $\lambda_1 >\!\!\!> \lambda_2$; “Corner”
- $\lambda_1 >\!\!\!> \lambda_2$; “Edge”
- $\lambda_1$ and $\lambda_2$ are small; $E$ is almost constant in all directions
- “Flat” region

$\lambda_1$ and $\lambda_2$ are small; $E$ is almost constant in all directions
Corner response function

\[ R = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2 \]

\( \alpha \): constant (0.04 to 0.06)
The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix $M$ in a Gaussian window around each pixel:

$$M = \begin{bmatrix}
\sum_{x,y} w(x, y) I_x^2 & \sum_{x,y} w(x, y) I_x I_y \\
\sum_{x,y} w(x, y) I_x I_y & \sum_{x,y} w(x, y) I_y^2
\end{bmatrix}$$

The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix $M$ in a Gaussian window around each pixel
3. Compute corner response function $R$

Harris Detector: Steps

Compute corner response $R$
The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix $M$ in a Gaussian window around each pixel
3. Compute corner response function $R$
4. Threshold $R$
5. Find local maxima of response function (nonmaximum suppression)

Harris Detector: Steps

Find points with large corner response: $R > \text{threshold}$
Harris Detector: Steps

Take only the points of local maxima of $R$
Harris Detector: Steps
Robustness of corner features

- What happens to corner features when the image undergoes geometric or photometric transformations?
Affine intensity change

- Only derivatives are used \( \Rightarrow \) invariance to intensity shift \( I \rightarrow I + b \)
- Intensity scaling: \( I \rightarrow a I \)

Partially invariant to affine intensity change
Image translation

- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation
Image rotation

Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation
Scaling

Corner

Corner location is not covariant w.r.t. scaling!

All points will be classified as edges