Generative adversarial networks: Introduction
Outline

• Generative modeling tasks
• Original GAN formulation
• Alternative GAN objectives
• Evaluating GANs
Generative modeling tasks

- **Generation**: learn to sample from the distribution represented by the training set
  - *Unsupervised learning task*
Generative modeling tasks

- Generation conditioned on class label or text prompt

Figure source
Generative modeling tasks

- Generation conditioned on image (*image-to-image translation*)

Designing a network for generative tasks

1. We need an architecture that can generate an image
   • Recall upsampling architectures for dense prediction
Designing a network for generative tasks

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Designing a network for generative tasks

1. We need an architecture that can generate an image
   • Recall upsampling architectures for dense prediction
2. We need to design the right loss function and training framework
Learning to sample

Training data $x \sim p_{\text{data}}$  
Generated samples $x \sim p_{\text{model}}$

We want to learn $p_{\text{model}}$ that matches $p_{\text{data}}$

Adapted from Stanford CS231n
Generative adversarial networks

- Train two networks with opposing objectives:
  - **Generator**: learns to generate samples
  - **Discriminator**: learns to distinguish between generated and real samples

GAN objective

• The discriminator $D(x)$ should output the probability that the sample $x$ is *real*
• That is, we want $D(x)$ to be close to 1 for real data and close to 0 for fake
• Expected conditional log likelihood for real and generated data:

$$
\mathbb{E}_{x \sim p_{\text{data}}} \log D(x) + \mathbb{E}_{x \sim p_{\text{gen}}} \log(1 - D(x))
= \mathbb{E}_{x \sim p_{\text{data}}} \log D(x) + \mathbb{E}_{z \sim p} \log(1 - D(G(z)))
$$

We seed the generator with noise $z$ drawn from a simple distribution $p$ (Gaussian or uniform)
GAN objective

\[ V(G,D) = \mathbb{E}_{x \sim p_{\text{data}}} \log D(x) + \mathbb{E}_{z \sim p} \log(1 - D(G(z))) \]

• The discriminator wants to correctly distinguish real and fake samples:
  \[ D^* = \arg \max_D V(G,D) \]

• The generator wants to fool the discriminator:
  \[ G^* = \arg \min_G V(G,D) \]

• Train the generator and discriminator jointly in a minimax game
GAN objective: Theoretical properties

\[ V(G, D) = \mathbb{E}_{x \sim p_{\text{data}}} \log D(x) + \mathbb{E}_{z \sim p} \log(1 - D(G(z))) \]

- Assuming unlimited capacity for generator and discriminator and unlimited training data:
  - The objective \( \min_G \max_D V(G, D) \) is equivalent to Jensen-Shannon divergence between \( p_{\text{data}} \) and \( p_{\text{gen}} \) and global optimum (Nash equilibrium) is given by \( p_{\text{data}} = p_{\text{gen}} \)
  - If at each step, \( D \) is allowed to reach its optimum given \( G \), and \( G \) is updated to decrease \( V(G, D) \), then \( p_{\text{gen}} \) with eventually converge to \( p_{\text{data}} \)
GAN training

\[ V(G, D) = \mathbb{E}_{x \sim p_{data}} \log D(x) + \mathbb{E}_{z \sim p} \log(1 - D(G(z))) \]

- Alternate between
  - Gradient ascent on discriminator:
    \[ D^* = \arg \max_D V(G, D) \]
  - Gradient descent on generator (minimize log-probability of generator samples being labeled “fake”):
    \[ G^* = \arg \min_G V(G, D) = \arg \min_G \mathbb{E}_{z \sim p} \log(1 - D(G(z))) \]
- In practice, do gradient ascent on generator (maximize log-probability of generator samples being labeled “real”):
  \[ G^* = \arg \max_G \mathbb{E}_{z \sim p} \log(D(G(z))) \]
Non-saturating GAN loss (NSGAN)

\[
\min_{w_G} \mathbb{E}_{z \sim p} \log(1 - D(G(z))) \quad \text{vs.} \quad \max_{w_G} \mathbb{E}_{z \sim p} \log(D(G(z)))
\]

Minimize log-probability of generator samples labeled “fake”

Maximize log-probability of generator samples labeled “real”
Non-saturating GAN loss (NSGAN)

\[
\min_{w_G} \mathbb{E}_{z \sim p} \log(1 - D(G(z))) \quad \text{vs.} \quad \max_{w_G} \mathbb{E}_{z \sim p} \log(D(G(z)))
\]

Minimize log-probability of generator samples labeled “fake”

Maximize log-probability of generator samples labeled “real”

Large gradients for low-quality samples

Small gradients for high-quality samples

Low discriminator score (low-quality samples)

High discriminator score (high-quality samples)

Want to learn from confidently rejected sample but gradients here are small

These samples already fool the discriminator so we don’t need large gradients here

Figure source
NSGAN training algorithm

- Update discriminator:
  - Repeat for $k$ steps:
    - Sample mini-batch of noise samples $z_1, ..., z_m$ and mini-batch of real samples $x_1, ..., x_m$
    - Update parameters of $D$ by stochastic gradient ascent on
      $$\frac{1}{m} \sum_m [\log D(x_m) + \log(1 - D(G(z_m)))]$$

- Update generator:
  - Sample mini-batch of noise samples $z_1, ..., z_m$
  - Update parameters of $G$ by stochastic gradient ascent on
    $$\frac{1}{m} \sum_m \log D(G(z_m))$$

- Repeat until happy with results
GAN: Schematic picture

- Update discriminator: push $D(x_{\text{data}})$ close to 1 and $D(G(z))$ close to 0
- The generator is a “black box” to the discriminator
GAN: Schematic picture

- Update generator: increase $D(G(z))$
  - Requires back-propagating through the composed generator-discriminator network (i.e., the discriminator cannot be a black box)
  - The generator is exposed to real data only via the output of the discriminator and its gradients
GAN: Schematic picture

• Test time – the discriminator is discarded
Original GAN results

MNIST digits

Toronto Face Dataset

Nearest real image for sample to the left

Original GAN results

DCGAN

• Early, influential convolutional architecture for generator

DCGAN

- Early, influential convolutional architecture for generator
- Discriminator architecture (empirically determined to give best training stability):
  - Don’t use pooling, only strided convolutions
  - Use Leaky ReLU activations (sparse gradients cause problems for training)
  - Use only one FC layer before the softmax output
  - Use batch normalization after most layers (in the generator also)

DCGAN results

Generated bedrooms after one epoch
DCGAN results

Generated bedrooms after five epochs
DCGAN results

More bedrooms

Notice repetition artifacts (analysis)

Source: F. Fleuret
DCGAN results

Interpolation between different points in the z space
DCGAN results

- Vector arithmetic in the z space
DCGAN results

- Vector arithmetic in the z space
DCGAN results

- Pose transformation by adding a “turn” vector
Problems with GAN training

- Stability
  - Parameters can oscillate or diverge, generator loss does not correlate with sample quality
  - Behavior very sensitive to hyperparameter selection
Problems with GAN training

• Mode collapse
  • Generator ends up modeling only a small subset of the training data
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• Alternative GAN objectives
Wasserstein GAN (WGAN)

- Motivated by *Wasserstein or Earth mover’s distance*, which is an alternative to JS divergence for comparing distributions
  - In practice, use linear activation instead of sigmoid in the discriminator and drop the logs from the objective:
    \[
    \min_G \max_D [\mathbb{E}_{x \sim p_{data}} D(x) - \mathbb{E}_{z \sim p} D(G(z))] 
    \]
  - Due to theoretical considerations, important to ensure smoothness of discriminator
  - This paper’s suggested method is clipping weights to fixed range \([-c, c]\)

M. Arjovsky, S. Chintala, L. Bottou, *Wasserstein generative adversarial networks*, ICML 2017
Wasserstein GAN (WGAN)

• Benefits (claimed)
  • Better gradients, more stable training

M. Arjovsky, S. Chintala, L. Bottou, Wasserstein generative adversarial networks, ICML 2017
Wasserstein GAN (WGAN)

- **Benefits (claimed)**
  - Better gradients, more stable training
  - Objective function value is more meaningfully related to quality of generator output

M. Arjovsky, S. Chintala, L. Bottou, *Wasserstein generative adversarial networks*, ICML 2017
Improved Wasserstein GAN (WGAN-GP)

• Weight clipping leads to problems with discriminator training
• Improved Wasserstein discriminator loss:

\[ \mathbb{E}_{\tilde{x} \sim p_{\text{gen}}} D(\tilde{x}) - \mathbb{E}_{x \sim p_{\text{real}}} D(x) \]

\[ + \lambda \mathbb{E}_{\tilde{x} \sim p_{\tilde{x}}} \left[ (\|\nabla_{\tilde{x}} D(\tilde{x})\|_2 - 1)^2 \right] \]

Unit norm gradient penalty on points \( \tilde{x} \) obtained by interpolating real and generated samples

I. Gulrajani, F. Ahmed, M. Arjovsky, V. Dumoulin, A. Courville, Improved training of Wasserstein GANs, NIPS 2017
## Improved Wasserstein GAN: Results

<table>
<thead>
<tr>
<th>DCGAN</th>
<th>LSGAN</th>
<th>WGAN (clipping)</th>
<th>WGAN-GP (ours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline ($G$: DCGAN, $D$: DCGAN)</td>
<td><img src="image" alt="Baseline" /></td>
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<tr>
<td>$G$: No BN and a constant number of filters, $D$: DCGAN</td>
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<tr>
<td>$G$: 4-layer 512-dim ReLU MLP, $D$: DCGAN</td>
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<td>No normalization in either $G$ or $D$</td>
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<td>Gated multiplicative nonlinearities everywhere in $G$ and $D$</td>
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<tr>
<td>tanh nonlinearities everywhere in $G$ and $D$</td>
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<tr>
<td>101-layer ResNet $G$ and $D$</td>
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</table>

Least Squares GAN (LSGAN)

- Use least squares cost for generator and discriminator
  - Equivalent to minimizing Pearson $\chi^2$ divergence

$$L_D = \mathbb{E}_{x \sim p_{\text{data}}} (D(x) - 1)^2 + \mathbb{E}_{z \sim p} (D(G(z)))^2$$

  - Push discrimin. response on real data close to 1
  - Push response on generated data close to 0

$$L_G = \mathbb{E}_{z \sim p} (D(G(z)) - 1)^2$$

  - Push response on generated data close to 1

Least Squares GAN (LSGAN)

- Benefits (claimed)
  - Higher-quality images

Least Squares GAN (LSGAN)

• Benefits (claimed)
  • Higher-quality images
  • More stable and resistant to mode collapse

GAN with hinge loss

- **Discriminator**: Drive discriminator score on real data above 1, on generated data below −1

\[
L_D = -\mathbb{E}_{x \sim p_{\text{data}}} \left[ \min(0, D(x) - 1) \right] - \mathbb{E}_{z \sim p} \left[ \min(0, -D(G(z)) - 1) \right]
\]

- **Generator**: maximize discriminator score on generated data

\[
L_G = -\mathbb{E}_{z \sim p} D(G(z))
\]

T. Miyato et al., *Spectral normalization for generative adversarial networks*, ICLR 2018
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How to evaluate GANs?

• Showing pictures of samples is not enough, especially for simpler datasets like MNIST, CIFAR, faces, bedrooms, etc.
• We cannot directly compute the likelihoods of high-dimensional samples (real or generated), or compare their distributions
• Many GAN approaches claim mainly to improve stability, which is hard to evaluate
GAN evaluation: Human studies

- Example: Turing test

GAN evaluation: Inception score (IS)

• Key idea: generators should produce images with a variety of recognizable object classes
  • Pass generated samples $x$ through an image classifier (InceptionNet), compute posterior class distributions $P(y|x)$ and marginal distribution $P(y)$
  • Compute Inception score as
    \[
    IS(G) = \exp[\mathbb{E}_{x \sim G} KL(P(y|x) \parallel P(y))].
    \]
• IS should be high when:
  • Samples $x$ contain recognizable objects, so entropy of $P(y|x)$ is low
  • The predicted labels of samples are diverse, so the entropy of $P(y)$ is high
  • What if a generator overfits (memorizes the training set)?
  • What if it outputs a single image per class?

GAN evaluation: Inception score (IS)

- Disadvantages
  - A GAN that simply memorizes the training data (overfitting) or outputs a single image per class (mode dropping) could still score well
  - Is sensitive to network weights, not necessarily valid for generative models not trained on ImageNet, can be gamed (Barratt & Sharma 2018)

*Figure 1. Sample of generated images achieving an Inception Score of 900.15. The maximum achievable Inception Score is 1000, and the highest achieved in the literature is on the order of 10.*
GAN evaluation: Fréchet Inception Distance (FID)

- **Key idea:** fit simple distributions (Gaussians) to statistics of feature activations for real and generated data; estimate divergence parametrically
  - Pass generated samples through a network (InceptionNet), compute activations for a chosen layer
  - Estimate multivariate mean and covariance of activations, compute *Fréchet distance* to those of real data
- **Advantages:** correlated with visual quality of samples and human judgment, can detect mode dropping (unlike IS)
- **Disadvantages:** cannot detect overfitting (like IS), can be sensitive to resampling and compression ([Parmar et al. 2021](#))

M. Heusel, H. Ramsauer, T. Unterthiner, B. Nessler, S. Hochreiter, *GANs trained by a two time-scale update rule converge to a local Nash equilibrium*, NIPS 2017
Are GANs created equal?

• From the abstract:

“We find that most models can reach similar scores with enough hyperparameter optimization and random restarts. This suggests that improvements can arise from a higher computational budget and tuning more than fundamental algorithmic changes … We did not find evidence that any of the tested algorithms consistently outperforms the non-saturating GAN introduced in Goodfellow et al. (2014)”

M. Lucic, K. Kurach, M. Michalski, O. Bousquet, S. Gelly, Are GANs created equal? A large-scale study, NIPS 2018