Image alignment
Alignment: Fitting of transformations

• Previously: fitting a model to features in one image

- Given: points $x_1, \ldots, x_n$
- Find: model $M$ that minimizes

$$\sum_i \text{residual}(x_i, M)$$
Alignment: Fitting of transformations

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  - Given: points $x_1, \ldots, x_n$
  - Find: model $M$ that minimizes
    $$\sum_i \text{residual}(x_i, M)$$

- Alignment: fitting a model to a transformation between pairs of features (matches) in two images
  - Given: matches $(x_1, x'_1), \ldots, (x_n, x'_n)$
  - Find: transformation $T$ that minimizes
    $$\sum_i \text{residual}(T(x_i), x'_i)$$
Alignment: Overview

• Motivation
• Fitting of transformations
  • Affine transformations
  • Homographies
• Robust alignment
  • Descriptor-based feature matching
  • RANSAC
• Large-scale alignment
  • Inverted indexing
  • Vocabulary trees
Alignment applications: Panorama stitching

http://matthewalunbrown.com/autostitch/autostitch.html
Alignment applications: Instance recognition

Model images

Test image

David G. Lowe. Distinctive image features from scale-invariant keypoints. IJCV 60 (2), pp. 91-110, 2004
Alignment applications: Instance recognition

T. Weyand and B. Leibe, Visual landmark recognition from Internet photo collections: A large-scale evaluation, CVIU 2015
Alignment applications: Large-scale reconstruction

Colosseum: 2,097 images, 819,242 points

Trevi Fountain: 1,935 images, 1,055,153 points

Pantheon: 1,032 images, 530,076 points

Hall of Maps: 275 images, 230,182 points

S. Agarwal et al. Building Rome in a Day. ICCV 2009
Feature-based alignment

• Find a set of feature matches that agree in terms of:
  a) Local appearance
  b) Geometric configuration
Feature-based alignment *really works!*

Source: N. Snavely
Feature-based alignment *really works*

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Alignment: Overview

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Alignment: Fitting of transformations

- Given: matches \((x_1, x'_1), \ldots, (x_n, x'_n)\)
- Find: transformation \(T\) that minimizes

\[
\sum_i \text{residual}(T(x_i), x'_i)
\]
2D transformation models

- Similarity (translation, scale, rotation)
- Affine
- Projective (homography)
Recall: Rotation

\[
\begin{pmatrix}
  x' \\
  y'
\end{pmatrix} = \begin{bmatrix}
  \cos(\theta) & -\sin(\theta) \\
  \sin(\theta) & \cos(\theta)
\end{bmatrix} \begin{pmatrix}
  x \\
  y
\end{pmatrix}
\]

- Estimating the rotation matrix requires enforcing nonlinear constraints, so we will skip the details.
2D transformation models

- Similarity (translation, scale, rotation)
- Affine
- Projective (homography)
Let’s start with affine transformations

• Simple fitting procedure: linear least squares
• Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras
• Can be used to initialize fitting for more complex models
Fitting an affine transformation

• Assume we know the correspondences, how do we get the transformation?

\[
\begin{pmatrix} x'_i \\ y'_i \end{pmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}
\]

\[ x'_i \quad M \quad x_i \quad t \]

Want to find \( M, t \) to minimize

\[
\sum_{i=1}^{n} \|x'_i - Mx_i - t\|^2
\]
Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?

\[(x'_i, y'_i) = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} (x_i, y_i) + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}\]

\[
\begin{pmatrix}
\begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \\
\begin{pmatrix} t_1 \\ t_2 \end{pmatrix}
\end{pmatrix}
= \begin{pmatrix} x'_i \\ y'_i \end{pmatrix}
\]
Fitting an affine transformation

• How many matches do we need to solve for the transformation parameters?

\[
\begin{bmatrix}
  x_i & y_i & \ldots & 0 & 0 & 1 & 0 \\
  0 & 0 & x_i & y_i & 0 & 1 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
  \end{bmatrix}
\begin{pmatrix}
  m_1 \\
  m_2 \\
  m_3 \\
  m_4 \\
  t_1 \\
  t_2 \\
  \vdots \\
\end{pmatrix} =
\begin{pmatrix}
  x'_i \\
  y'_i \\
  \vdots \\
\end{pmatrix}
\]
Fitting a homography

• A *homography* is a plane projective transformation (transformation taking a quad to another arbitrary quad)
Homography in the real world

• The transformation between two views of a planar surface

• The transformation between images from two cameras that share the same center
Application: Panorama stitching

Source: Hartley & Zisserman
Fitting a homography

- Recall:

\[ x'(x, y) = \frac{ax + by + c}{gx + hy + i'}, \quad y'(x, y) = \frac{dx + ey + f}{gx + hy + i} \]
Last time: Alignment

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Fitting a homography

• We need 2D *homogeneous coordinates*:

\[
\begin{pmatrix}
  x \\
  y \\
  1
\end{pmatrix} \leftrightarrow \begin{pmatrix}
  x \\
  y
\end{pmatrix} \quad (x, y) \Rightarrow \begin{pmatrix}
  x \\
  y \\
  w
\end{pmatrix} \leftrightarrow \begin{pmatrix}
  x/w \\
  y/w
\end{pmatrix}
\]

Converting to homogeneous coordinates \quad Converting from homogeneous coordinates

• All homogeneous coordinate vectors that are scalar multiples of each other represent the same point!

• Equation for homography in homogeneous coordinates:

\[
\begin{pmatrix}
  x' \\
  y' \\
  1
\end{pmatrix} = \begin{bmatrix}
  h_{11} & h_{12} & h_{13} \\
  h_{21} & h_{22} & h_{23} \\
  h_{31} & h_{32} & h_{33}
\end{bmatrix} \begin{pmatrix}
  x \\
  y \\
  1
\end{pmatrix} \quad x' \approx Hx
\]

“equal up to scale”
Fitting a homography

• Constraint from a match \((x_i, x'_i)\): \(x'_i \cong Hx_i\)
• How can we get rid of the scale ambiguity?
• Cross product trick: \(x'_i \times Hx_i = 0\)
• Recall cross product:

\[
\begin{pmatrix}
  a \\
  b \\
  c
\end{pmatrix} \times 
\begin{pmatrix}
  a' \\
  b' \\
  c'
\end{pmatrix} =
\begin{pmatrix}
  bc' - b'c \\
  ca' - c'a \\
  ab' - a'b
\end{pmatrix}
\]

• Let \(h^T_1, h^T_2, h^T_3\) be the rows of \(H\). Then

\[
x'_i \times Hx_i =
\begin{pmatrix}
  x'_i \\
  y'_i \\
  1
\end{pmatrix} \times 
\begin{pmatrix}
  h^T_1 x_i \\
  h^T_2 x_i \\
  h^T_3 x_i
\end{pmatrix} =
\begin{pmatrix}
  y'_i h^T_3 x_i - h^T_2 x_i \\
  h^T_1 x_i - x'_i h^T_3 x_i \\
  x'_i h^T_2 x_i - y'_i h^T_1 x_i
\end{pmatrix}
\]
Fitting a homography

- Constraint from a match \((x_i, x'_i)\):

\[
x'_i \times Hx_i = \begin{pmatrix} x'_i \\ y'_i \\ 1 \end{pmatrix} \times \begin{pmatrix} h^T_1 x_i \\ h^T_2 x_i \\ h^T_3 x_i \end{pmatrix} = \begin{pmatrix} y'_i h^T_3 x_i - h^T_2 x_i \\ h^T_1 x_i - x'_i h^T_3 x_i \\ x'_i h^T_2 x_i - y'_i h^T_1 x_i \end{pmatrix}
\]

- Rearranging the terms:

\[
\begin{bmatrix}
0^T & -x_i^T & y'_i x_i^T \\
x_i^T & 0^T & -x'_i x_i^T \\
-y'_i x_i^T & x'_i x_i^T & 0^T
\end{bmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = 0
\]

- Are these equations independent?
Fitting a homography

• Final linear system:

\[
\begin{bmatrix}
0^T & x_1^T & -y_1'x_1^T \\
x_1^T & 0^T & -x_1'x_1^T \\
\cdots & \cdots & \cdots \\
0^T & x_n^T & -y_n'x_n^T \\
x_n^T & 0^T & -x_n'x_n^T \\
\end{bmatrix}
\begin{pmatrix}
h_1 \\
h_2 \\
h_3 \\
\end{pmatrix} = 0 \quad Ah = 0
\]

• Homogeneous least squares: find \( h \) minimizing \( \|Ah\|^2 \)
  • Solution is eigenvector of \( A^TA \) corresponding to smallest eigenvalue
  • What is the minimum number of matches needed for a solution?
    • Four: \( A \) has 8 degrees of freedom (9 parameters, but scale is arbitrary), one match gives us two linearly independent equations
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Robust feature-based alignment

• So far, we’ve assumed that we are given a set of correspondences between the two images we want to align
• What if we don’t know the correspondences?
Robust feature-based alignment

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- What if we don’t know the correspondences?
Robust feature-based alignment
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- Extract features
Robust feature-based alignment

- Extract features
- Compute *putative matches*
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- Loop:
  - *Hypothesize* transformation $T$
Robust feature-based alignment

- Extract features
- Compute putative matches
- Loop:
  - Hypothesize transformation $T$
  - Verify transformation (search for other matches consistent with $T$)
Robust feature-based alignment

- Extract features
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  - Hypothesize transformation $T$
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Generating putative correspondences
Generating putative correspondences

- Need to compare feature descriptors of local patches surrounding interest points
Feature descriptors

• Recall: feature detection vs. feature description
Comparing feature descriptors

- Simplest descriptor: vector of raw intensity values
- How to compare two such vectors \( \mathbf{u} \) and \( \mathbf{v} \)?
  - **Sum of squared differences** (SSD):
    \[
    \text{SSD}(\mathbf{u}, \mathbf{v}) = \sum_i (u_i - v_i)^2
    \]
  - **Normalized correlation**: dot product between \( \mathbf{u} \) and \( \mathbf{v} \) normalized to have zero mean and unit norm:
    \[
    \rho(\mathbf{u}, \mathbf{v}) = \frac{\sum_i (u_i - \bar{u})(v_i - \bar{v})}{\sqrt{\left(\sum_j (u_j - \bar{u})^2\right)\left(\sum_j (v_j - \bar{v})^2\right)}}
    \]
  - Why would we prefer normalized correlation over SSD?
Disadvantage of intensity vectors as descriptors

- Small deformations can affect the matching score a lot
Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into $4 \times 4$ sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: $4 \times 4 \times 8 = 128$ dimensions

Feature descriptors: SIFT

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  • Resulting descriptor: $4 \times 4 \times 8 = 128$ dimensions

• What are the advantages of SIFT descriptor over raw pixel values?
  • Gradients are less sensitive to illumination change
  • Pooling of gradients over the sub-patches achieves robustness to small shifts, but still preserves some spatial information

Generating putative correspondences

- For each patch in one image, find a short list of patches in the other image that could match it based solely on appearance.
Rejection of ambiguous matches

• How can we tell which putative matches are more reliable?
• Heuristic: compare distance of nearest neighbor to that of second nearest neighbor
Rejection of ambiguous matches

• How can we tell which putative matches are more reliable?
• Heuristic: compare distance of nearest neighbor to that of second nearest neighbor
  • Ratio of closest distance to second-closest distance will be high for features that are not distinctive

Threshold of 0.8 found to provide good separation

Robust alignment

• Even after filtering out ambiguous matches, the set of putative matches still contains a very high percentage of outliers

• Solution: RANSAC

• RANSAC loop:
  1. Randomly select a seed group of matches
  2. Compute transformation from seed group
  3. Find inliers to this transformation
  4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

• At the end, keep the transformation with the largest number of inliers
RANSAC example: Translation

Putative matches
RANSAC example: Translation

Select one match, count inliers
RANSAC example: Translation

Select one match, count inliers
RANSAC example: Translation

Select translation with the most inliers
Alternative for robust alignment: Hough voting

- A single SIFT match can vote for translation, rotation, and scale parameters of a transformation between two images
  - Votes can be accumulated in a 4D Hough space with large bins
  - Clusters of matches falling into the same bin should undergo a more precise verification procedure

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Scalability: Alignment to large databases

- What if we need to align a test image with thousands or millions of images in a model database?
  - Efficient putative match generation: approximate descriptor similarity search, inverted indices
Large-scale visual search

Inverted indexing

Reranking/Geometric verification

Model images or exemplars

Input features in new image

Local feature descriptors from model images

Candidate matches based on descriptor similarity

Figure from: Kristen Grauman and Bastian Leibe, *Visual Object Recognition*, Synthesis Lectures on Artificial Intelligence and Machine Learning, April 2011, Vol. 5, No. 2, Pages 1-181
How to do the indexing?

- Idea: find a set of visual codewords to which descriptors can be quantized
Recall: Visual codebook for implicit shape models

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004
**K-means clustering**

- We want to find $K$ cluster centers and an assignment of points to cluster centers to minimize the sum of squared Euclidean distances between each point and its assigned cluster center:

$$\sum_i \sum_k a_{ik} \|x_i - c_k\|^2$$

- Sum over all points
- Sum over all clusters
- Point $x_i$ is assigned to cluster $k$
- Center of cluster $k$
K-means clustering

• We want to find $K$ cluster centers and an assignment of points to cluster centers to minimize the sum of squared Euclidean distances between each point and its assigned cluster center:

$$\sum_i \sum_k a_{ik} \| x_i - c_k \|^2$$

• Algorithm:
  • Randomly initialize $K$ cluster centers
  • Iterate until convergence:
    – Assign each data point to its nearest center
    – Recompute each cluster center as the mean of all points assigned to it
K-means example
How to do the indexing?

- Cluster descriptors in the database to form codebook
- At query time, quantize descriptors in query image to nearest codevectors
- Problem solved?
Efficient indexing technique: Vocabulary trees

D. Nistér and H. Stewénius, Scalable Recognition with a Vocabulary Tree, CVPR 2006
Hierarchical k-means clustering of descriptor space (vocabulary tree)

Slide credit: D. Nister
Vocabulary tree/inverted index
Populating the vocabulary tree/inverted index

Model images

Slide credit: D. Nister
Populating the vocabulary tree/inverted index
Populating the vocabulary tree/inverted index

Model images

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Populating the vocabulary tree/inverted index

Model images

Slide credit: D. Nister
Looking up a test image

Model images

Test image

Slide credit: D. Nister