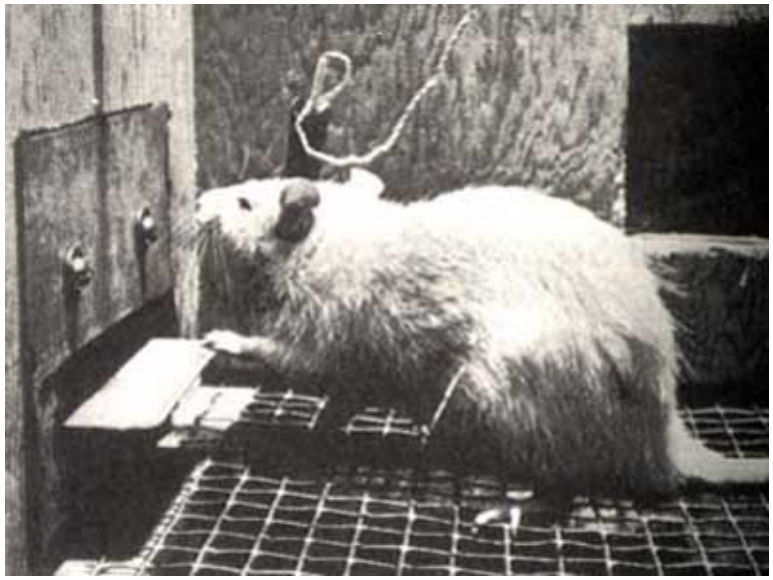
Introduction to deep reinforcement learning

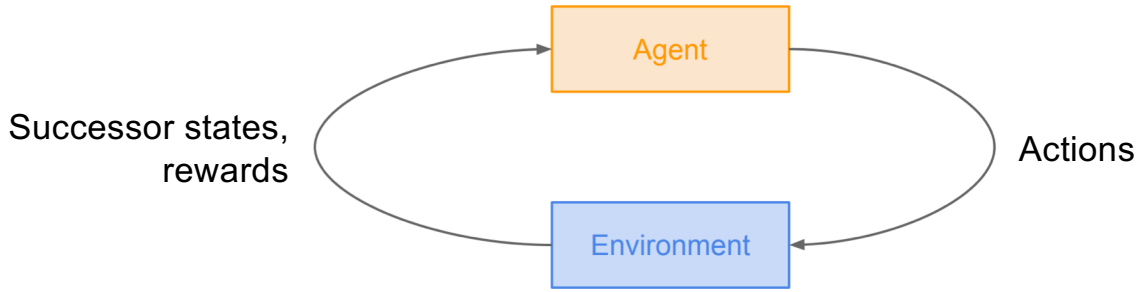


Outline

- Introduction to reinforcement learning
- Markov Decision Process (MDP) formalism
- The Bellman equation
- Q-learning
- Deep Q networks (DQN)
- Extensions
  - Double DQN
  - Dueling DQN

Reinforcement learning (RL)

- Setting: agent that can take *actions* affecting the *state* of the environment and observe occasional *rewards* that depend on the state
- Goal: learn a *policy* (mapping from states to actions) to maximize expected reward over time



RL vs. supervised learning

- **Reinforcement learning loop**
  - From state s, take action a determined by *policy* $\pi(s)$
  - Environment selects next state s' based on *transition model* $P(s'|s,a)$
  - Observe s' and reward $r(s')$, update policy

- **Supervised learning loop**
  - Get input x_i sampled i.i.d. from data distribution
  - Use model with parameters w to predict output y
  - Observe target output y_i and loss $l(w, x_i, y_i)$
  - Update w to reduce loss: $w \leftarrow w - \eta \nabla l(w, x_i, y_i)$

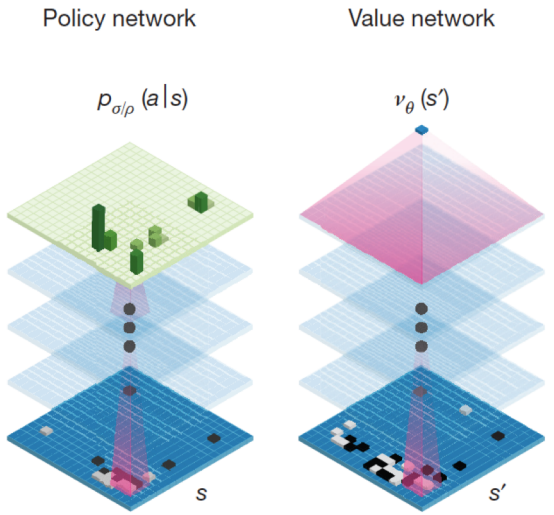
RL vs. supervised learning

- **Reinforcement learning**
  - Agent's actions affect the environment and help to determine next observation
  - Rewards may be sparse
  - Rewards are *not differentiable* w.r.t. model parameters

- **Supervised learning**
  - Next input does not depend on previous inputs or agent's predictions
  - There is a supervision signal at every step
  - Loss is differentiable w.r.t. model parameters

Example applications of deep RL

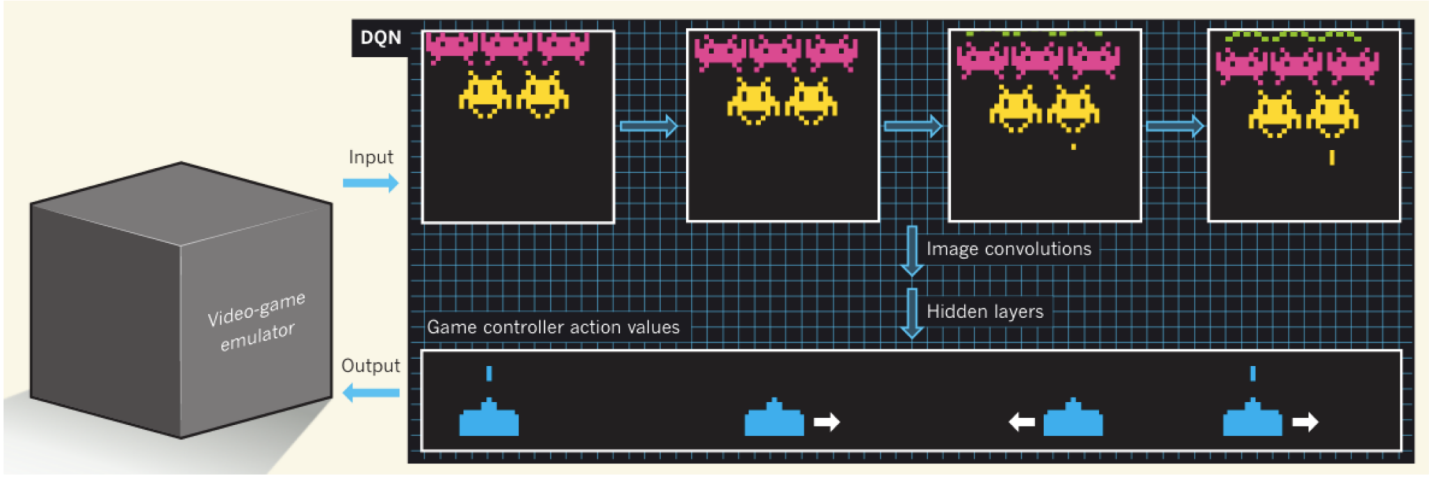
- AlphaGo and AlphaZero



https://deepmind.com/research/alphago/

Example applications of deep RL

- Playing video games



Video

V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller,
Human-level control through deep reinforcement learning, *Nature* 2015

Example applications of deep RL

- Sensorimotor learning

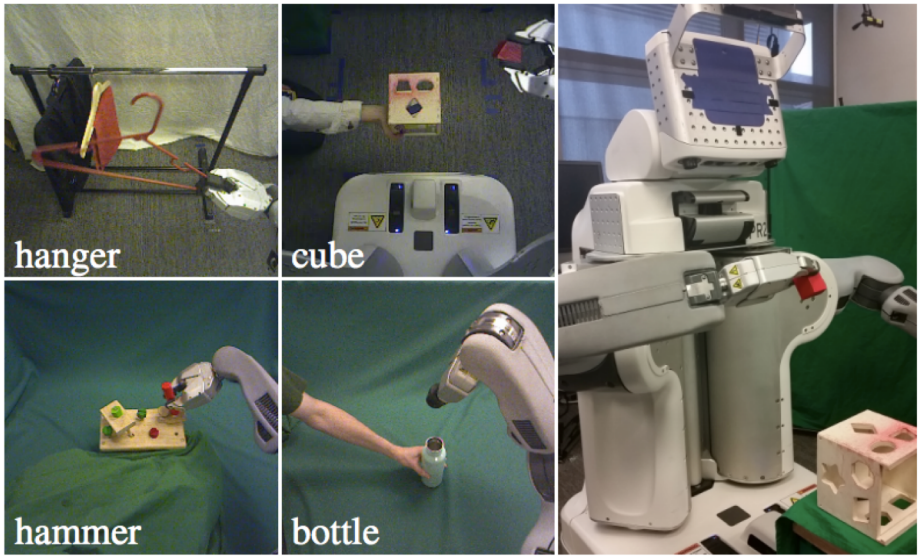


Fig. 1: Our method learns visuomotor policies that directly use camera image observations (left) to set motor torques on a PR2 robot (right).

Video

S. Levine, C. Finn, T. Darrell and P. Abbeel, End-to-End Training of Deep Visuomotor Policies, JMLR 2016

Example applications of deep RL

- Sensorimotor learning

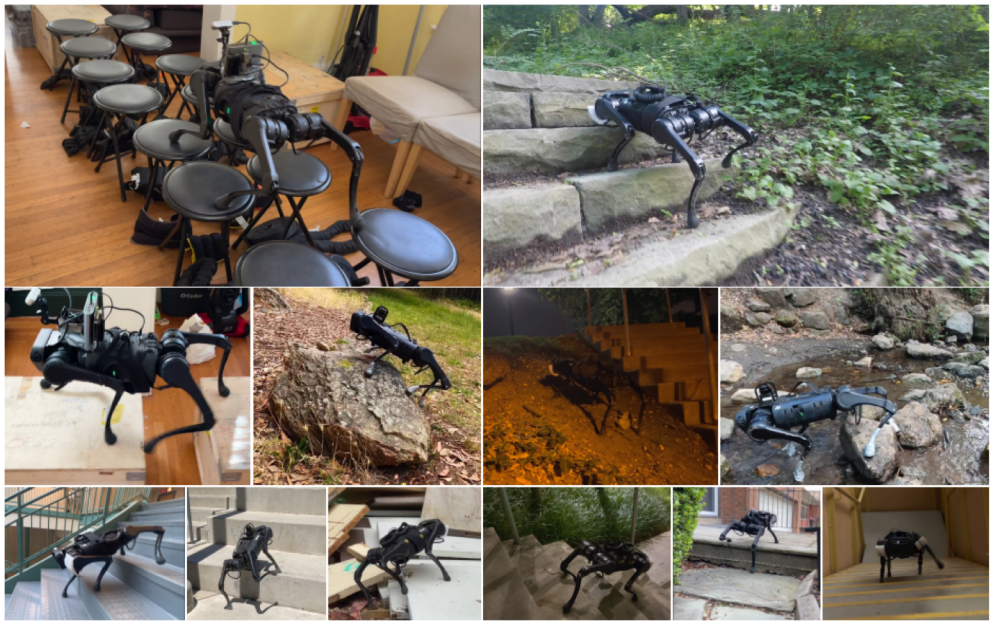
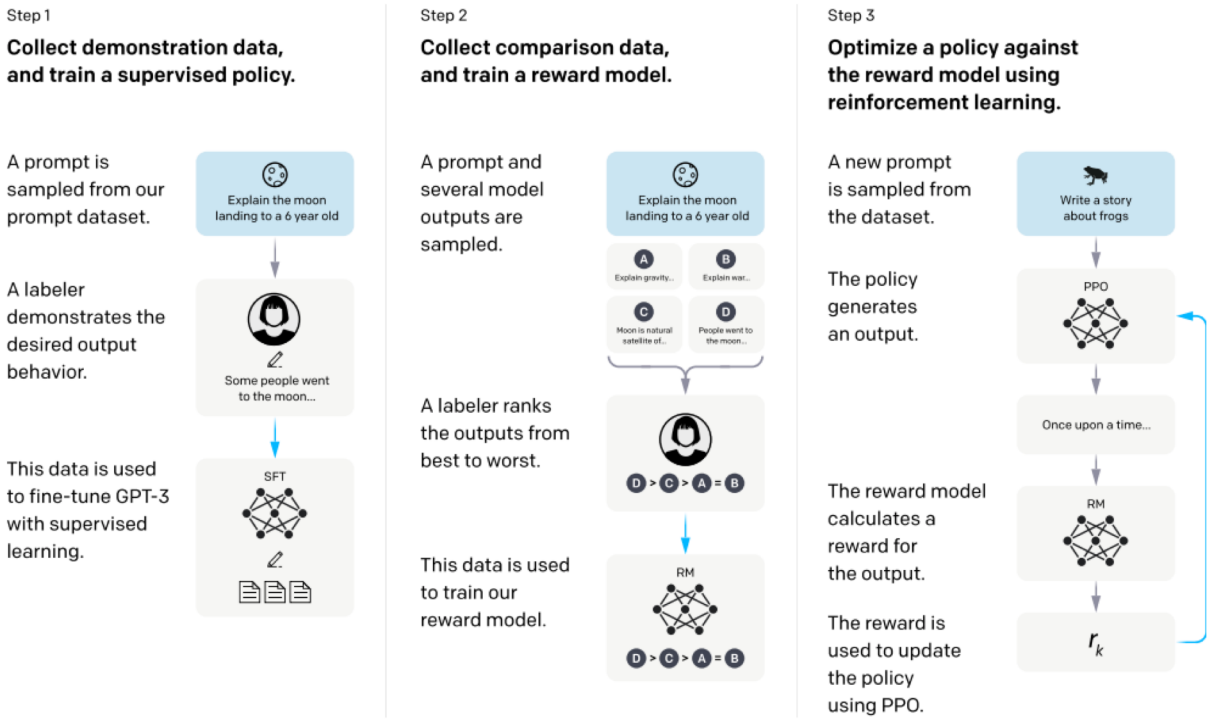


Figure 1: Our robot can traverse a variety of challenging terrain in indoor and outdoor environments, urban and natural settings during day and night using a single front-facing depth camera. The robot can traverse curbs, stairs and moderately rocky terrain. Despite being much smaller than other commonly used legged robots, it is able to climb stairs and curbs of a similar height. Videos at https://vision-locomotion.github.io

A. Agarwal, A. Kumar, J. Malik, and D. Pathak. Legged Locomotion in Challenging Terrains using Egocentric Vision. CoRL 2022

Example applications of deep RL

- Improving large language models

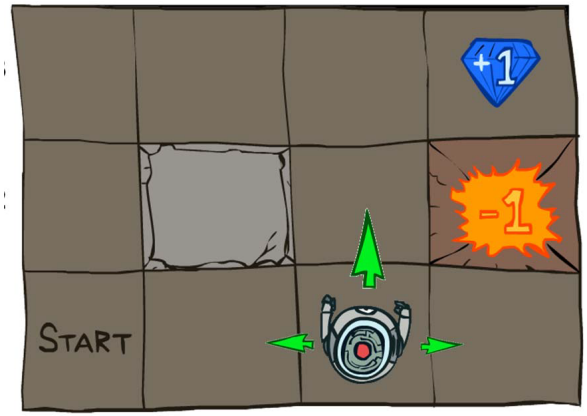


L. Ouyang et al. Training language models to follow instructions with human feedback. NeurIPS 2022

Formalism: Markov Decision Processes

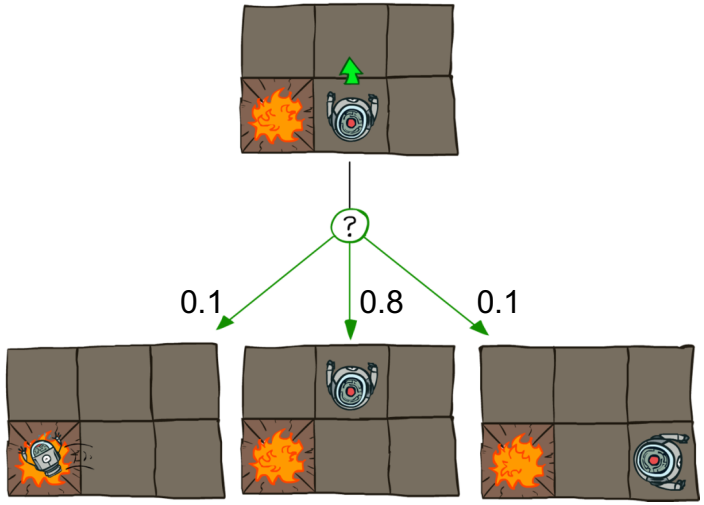
- Components:
  - **States** s, beginning with initial state s_0
  - **Actions** a
  - **Transition model** $P(s' \mid s, a)$
    - *Markov assumption*: the probability of going to s' from s depends only on s and a and not on any other past actions or states
  - **Reward function** $r(s)$
- **Policy** $\pi(s)$: the action that an agent takes in any given state
  - The "solution" to an MDP

Example MDP: Grid world



$r(s) = -0.04$ for every non-terminal state

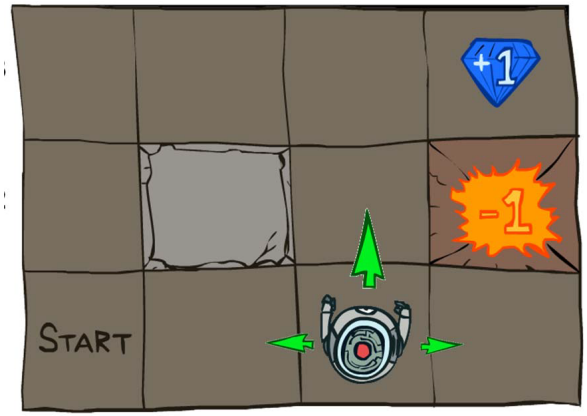
Transition model:

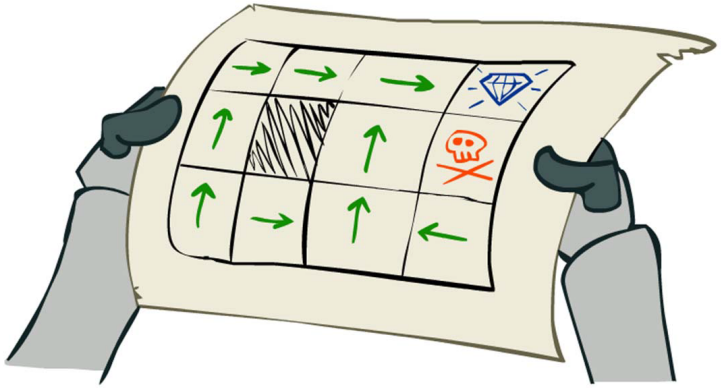


Source: P. Abbeel and D. Klein

Example MDP: Grid world

- Goal: find the best policy

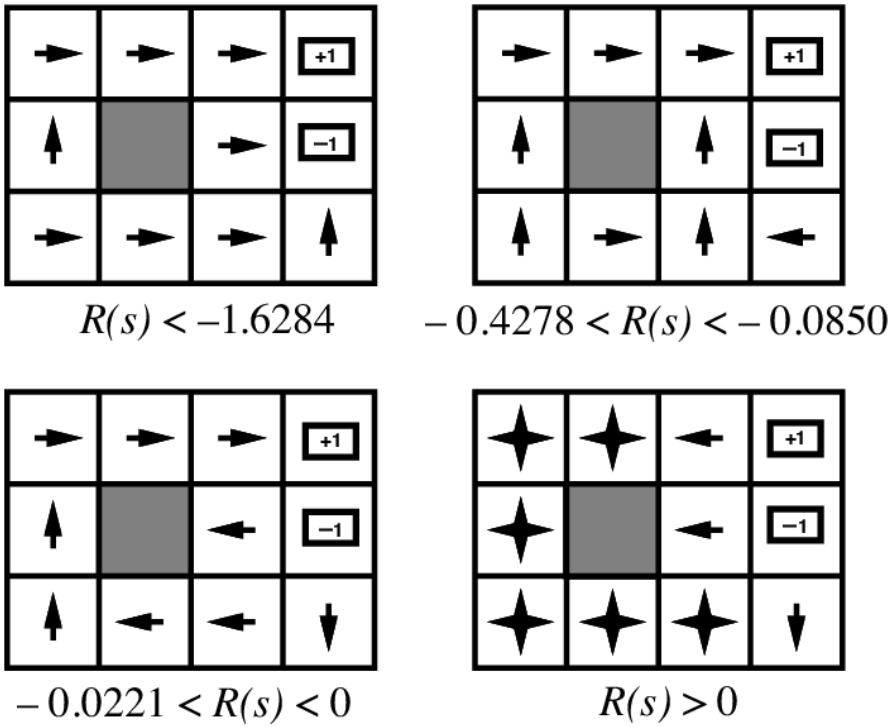




Source: P. Abbeel and D. Klein

Example MDP: Grid world

- Optimal policies for various values of $r(s)$:



Cumulative rewards of state sequences

- Suppose that following policy π starting in state s_0 leads to a *sequence* or *trajectory* $\tau = (s_0, s_1, s_2, \ldots)$
- How do we define the cumulative reward of this trajectory?
- **Problem:** the sum of rewards of individual states grows is not normalized w.r.t. sequence length and can even be infinite
- **Solution:** define cumulative reward as sum of rewards *discounted* by a factor γ, $0 < \gamma \leq 1$

Image source: P. Abbeel and D. Klein

Discounting

- *Discounted cumulative reward* of trajectory $\tau = (s_0, s_1, s_2, s_3, \dots)$:

$$r(s_0, s_1, s_2, s_3, \dots) = r(s_0) + \gamma\, r(s_1) + \gamma^2 r(s_2) + \gamma^3 r(s_3) + \cdots$$
$$= \sum_{t \geq 0} \gamma^t\, r(s_t)$$

- Sum is bounded by $\frac{r_{\max}}{1-\gamma}$ (assuming $0 < \gamma \leq 1$)
- Helps algorithms converge
- Notice:

$$r(s_0, s_1, s_2, s_3, \dots) = r(s_0) + \gamma\, r(s_1, s_2, s_3, \dots)$$

Cumulative reward of trajectory starting at s_0 | Reward at s_0 | Discounted reward of trajectory starting at s_1

Value function

- The *value function* $V^{\pi}(s)$ of a state s w.r.t. policy π is the expected cumulative reward of following that policy starting in s:

$$V^{\pi}(s) = \mathbb{E}_{\tau}[r(\tau)|\ s_0 = s, \pi]$$

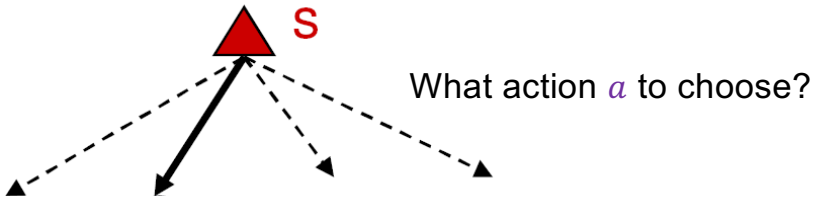
  where τ is a trajectory with starting state s, actions given by π, and successor states drawn according to transition model: $s_{t+1} \sim P(\cdot\,|s_t, a_t)$

- The *optimal value* of a state is the value achievable by following the best possible policy:

$$V^{*}(s) = \max_{\pi} V^{\pi}(s)$$

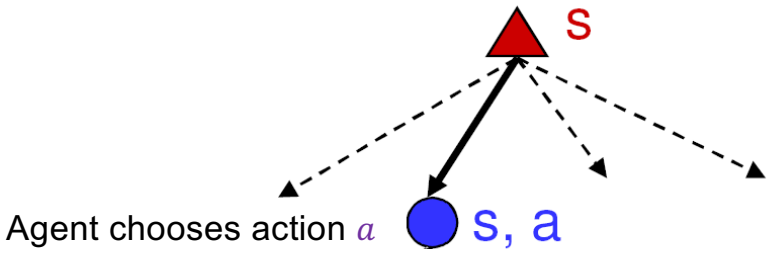
The optimal policy

- How do we express the optimal policy in terms of optimal state values?



The optimal policy

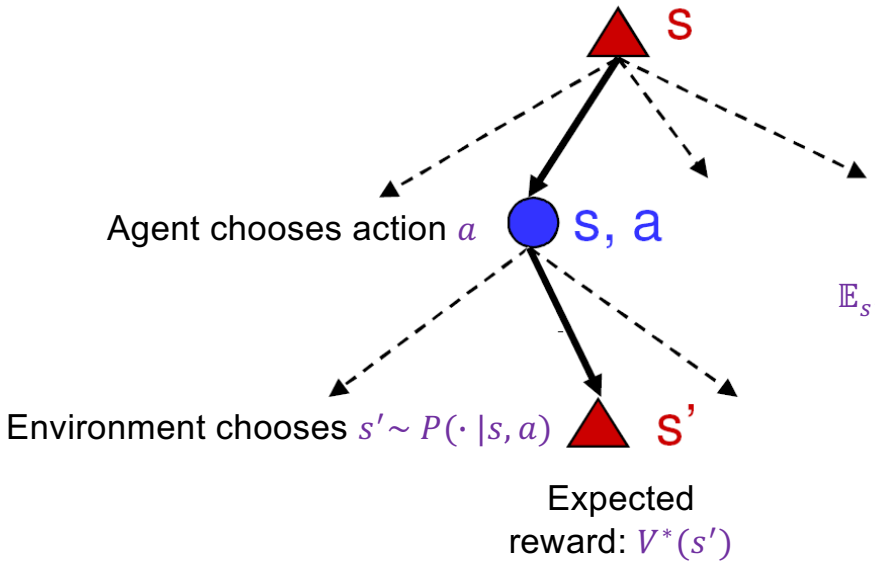
- How do we express the optimal policy in terms of optimal state values?



The optimal policy

- How do we express the optimal policy in terms of optimal state values?
  - Take the action that maximizes the expected future cumulative value:

$$\pi^*(s) = \arg\max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$



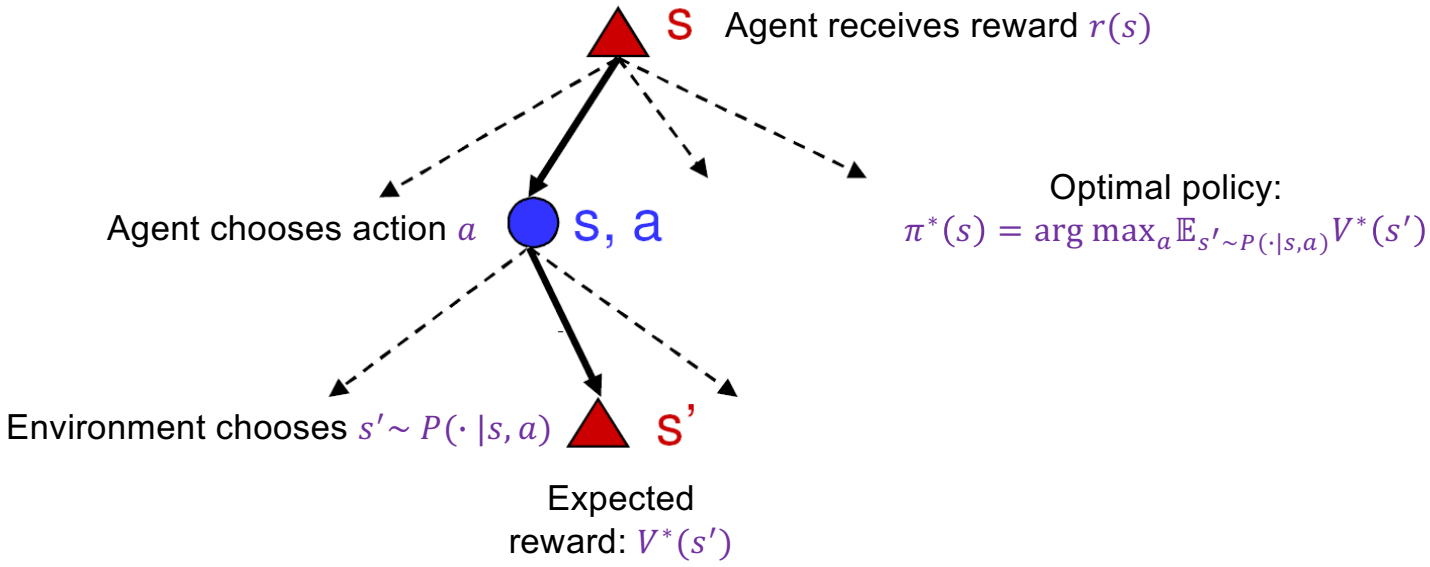
Expected value for action a:

$$\mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s') = \sum_{s'} P(s'|s,a) V^*(s')$$

The Bellman equation

- Recursive relationship between optimal values of successive states:

$$V^*(s) = r(s) + \gamma \max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$



The Bellman equation

- Recursive relationship between optimal values of successive states:

$$V^*(s) = r(s) + \gamma \max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$

Reward in current state

Discounted expected future reward assuming agent follows the optimal policy

Outline

- Introduction to reinforcement learning
- Markov Decision Process (MDP) formalism
- The Bellman equation
- Q-learning

Q-learning

- To choose actions using value functions, we need to know the transition model:

$$\pi^*(s) = \arg\max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$
$$= \arg\max_a \sum_{s'} P(s'|s,a) V^*(s')$$

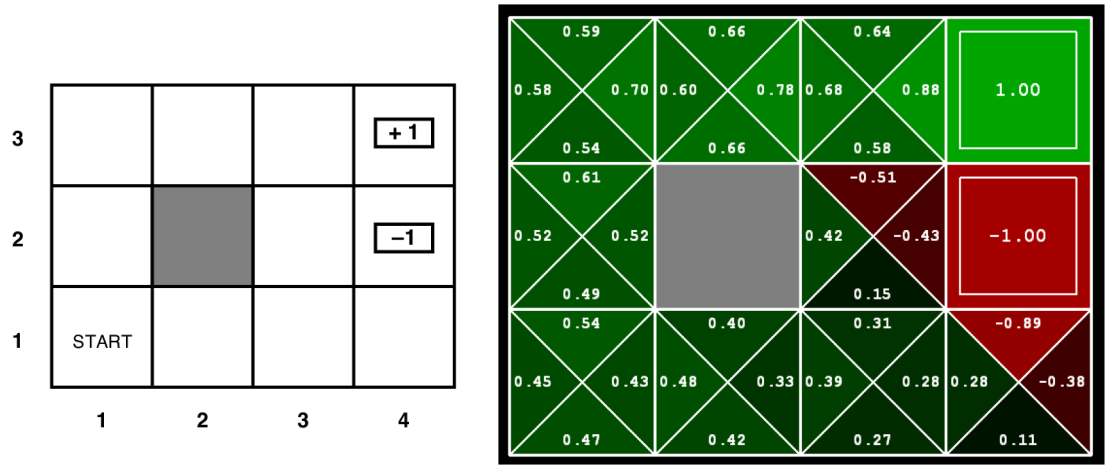
- It is more convenient to define the value of a *state-action pair*:

$$Q^\pi(s,a) = \mathbb{E}_\tau[r(\tau)|s_0 = s,\, a_0 = a, \pi]$$

- Then the optimal policy is given by

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

Q-value function: Example



Bellman equation for Q-values

- Relationship between regular values and Q-values:

$$V^*(s) = \max_a Q^*(s,a)$$

- Regular Bellman equation:

$$V^*(s) = r(s) + \gamma \max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$

- Bellman equation for Q-values:

$$\begin{aligned} Q^*(s,a) &= r(s) + \gamma \, \mathbb{E}_{s' \sim P(\cdot|s,a)} \max_{a'} Q^*(s',a') \\ &= \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[r(s) + \gamma \max_{a'} Q^*(s',a') \right] \end{aligned}$$

Finding the optimal policy

- The Bellman equation is a constraint on Q-values of successive states:

$$Q^*(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)}\left[r(s) + \gamma \max_{a'} Q^*(s',a') | s,a\right]$$

- We could think of $Q^*(s,a)$ as a table indexed by states and actions, and try to solve the system of Bellman equations to fill in the unknown values of the table
- **Problem:** state spaces for interesting problems are huge
- **Solution:** approximate Q-values using a parametric function:

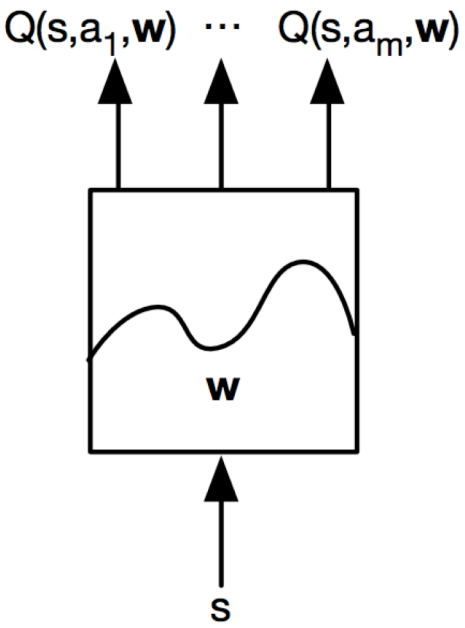
$$Q^*(s,a) \approx Q_w(s,a)$$

RL: Outline

- Introduction to reinforcement learning
- Markov Decision Process (MDP) formalism and classical Bellman equation
- Q-learning
- Deep Q networks

Deep Q-learning

- Train a deep neural network to estimate Q-values:



Source: D. Silver

V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller, Human-level control through deep reinforcement learning, *Nature* 2015

Deep Q-learning

$$Q^*(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)}\left[r(s) + \gamma \max_{a'} Q^*(s',a')|s,a\right]$$

- Idea: at each step of training, update model parameters w to "nudge" the left-hand side toward the right-hand "target":

$$y_{\text{target}}(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)}\left[r(s) + \gamma \max_{a'} Q_{\text{target}}(s',a')|s,a\right]$$

- Loss function:

$$L(w) = \mathbb{E}_{s,a}\left[(y_{\text{target}}(s,a) - Q_w(s,a))^2\right]$$

Deep Q-learning

- Target: $y_{\text{target}}(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)}\left[r(s) + \gamma \max_{a'} Q_{\text{target}}(s',a') | s,a\right]$
- Loss: $L(w) = \mathbb{E}_{s,a\sim\rho}\left[(y_{\text{target}}(s,a) - Q_w(s,a))^2\right]$

- Gradient update:

$$\nabla_w L(w) = \mathbb{E}_{s,a\sim\rho}\left[(y_{\text{target}}(s,a) - Q_w(s,a))\,\nabla_w Q_w(s,a)\right]$$
$$= \mathbb{E}_{s,a\sim\rho,s'}\left[(r(s) + \gamma \max_{a'} Q_{\text{target}}(s',a') - Q_w(s,a))\,\nabla_w Q_w(s,a)\right]$$

- SGD training: replace expectation by sampling transitions (s,a,s') using *behavior distribution* and *experience replay*

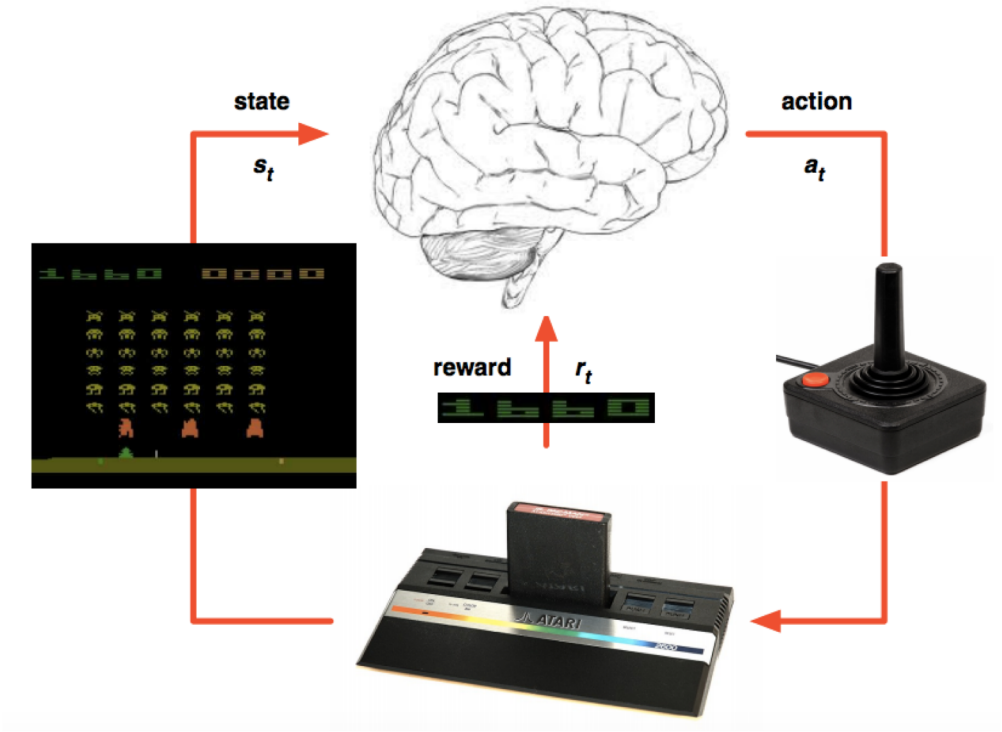
Deep Q-learning algorithm

- At each time step:
  - Take action a_t according to *epsilon-greedy policy*
  - Store experience $(s_t, a_t, r_{t+1}, s_{t+1})$ in *replay memory buffer*

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
…
$s_t, a_t, r_{t+1}, s_{t+1}$

  - Randomly sample *mini-batch* of experiences from the buffer
  - Perform gradient descent step on loss:
    $L(w) = \mathbb{E}_{s,a,s'}\left[(r(s) + \gamma \max_{a'} Q_{\text{target}}(s', a') - Q_w(s, a))^2\right]$
  - Update target network every C steps

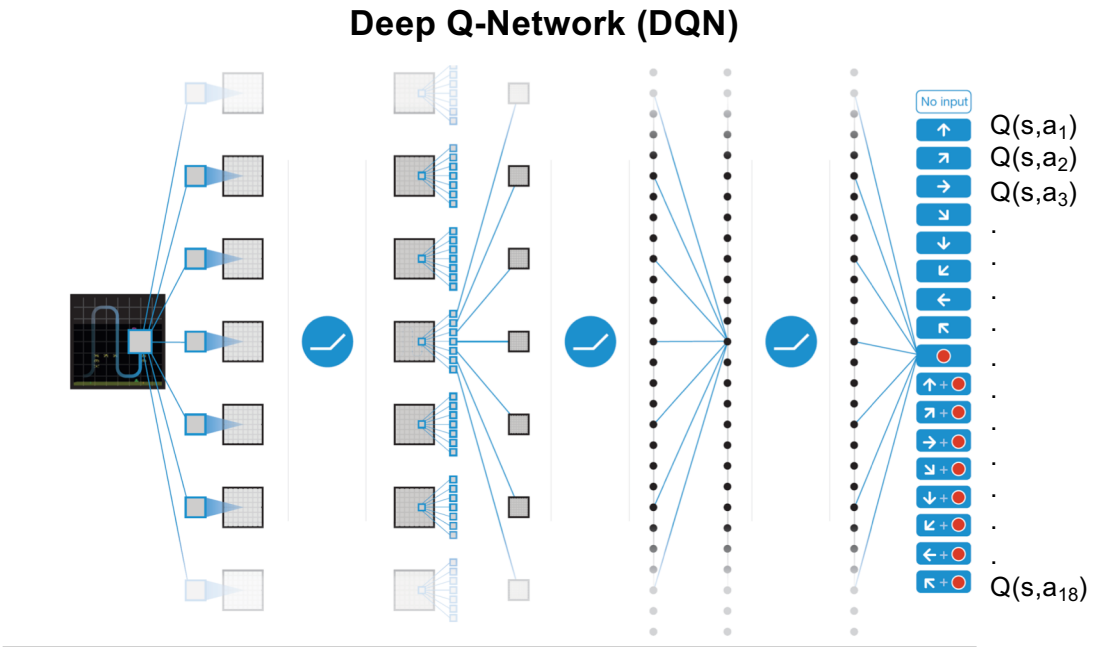
Deep Q-learning in Atari



V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller,
Human-level control through deep reinforcement learning, *Nature* 2015

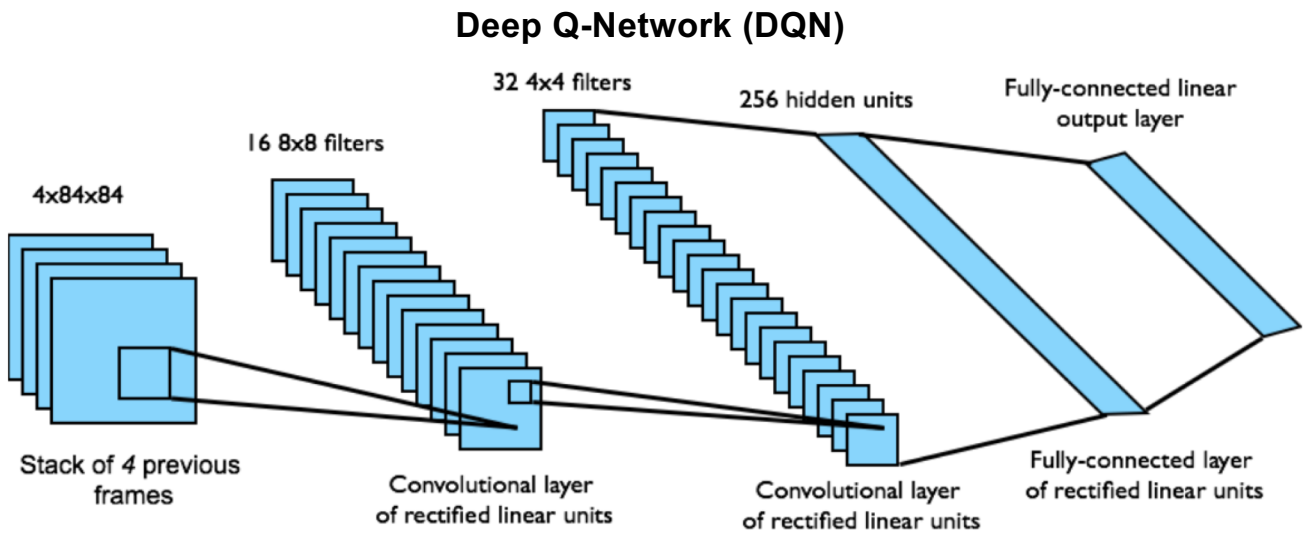
Deep Q-learning in Atari

- End-to-end learning of $Q(s,a)$ from pixels s
- Output is $Q(s,a)$ for 18 joystick/button configurations
- Reward is change in score for that step

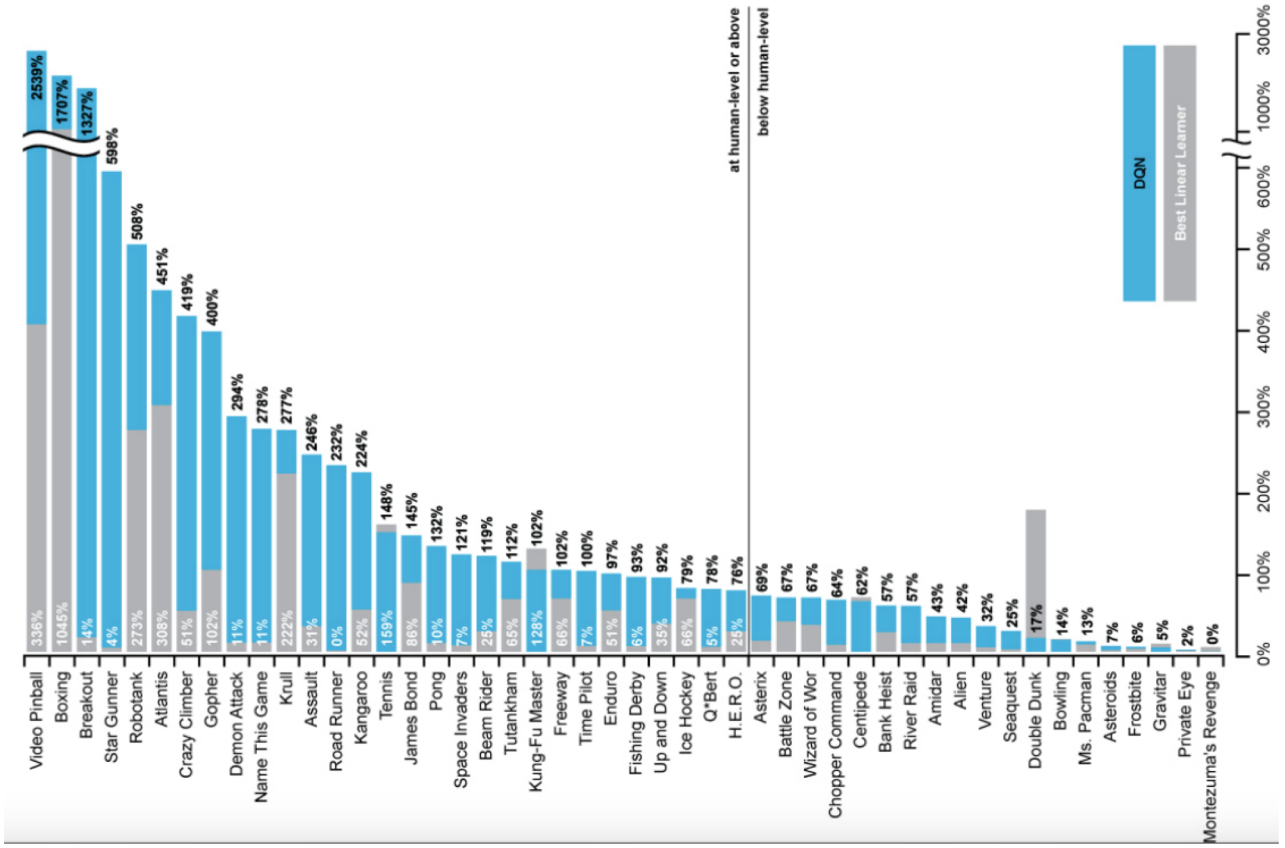


Deep Q-learning in Atari

- Input state is stack of raw pixels (grayscale) from last 4 frames
- Network architecture and hyperparameters fixed for all games



Deep Q-learning in Atari



Breakout demo

https://www.youtube.com/watch?v=TmPfTpjtdgg

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  - Dueling DQN

Extension: Double Q-learning

- Max operator in standard Q-learning is used both to select and evaluate an action, leading to systematic over-estimation of Q-values
- Modification: *select* action using the online (current) network, but *evaluate* Q-value using the target network
- Regular DQN target:

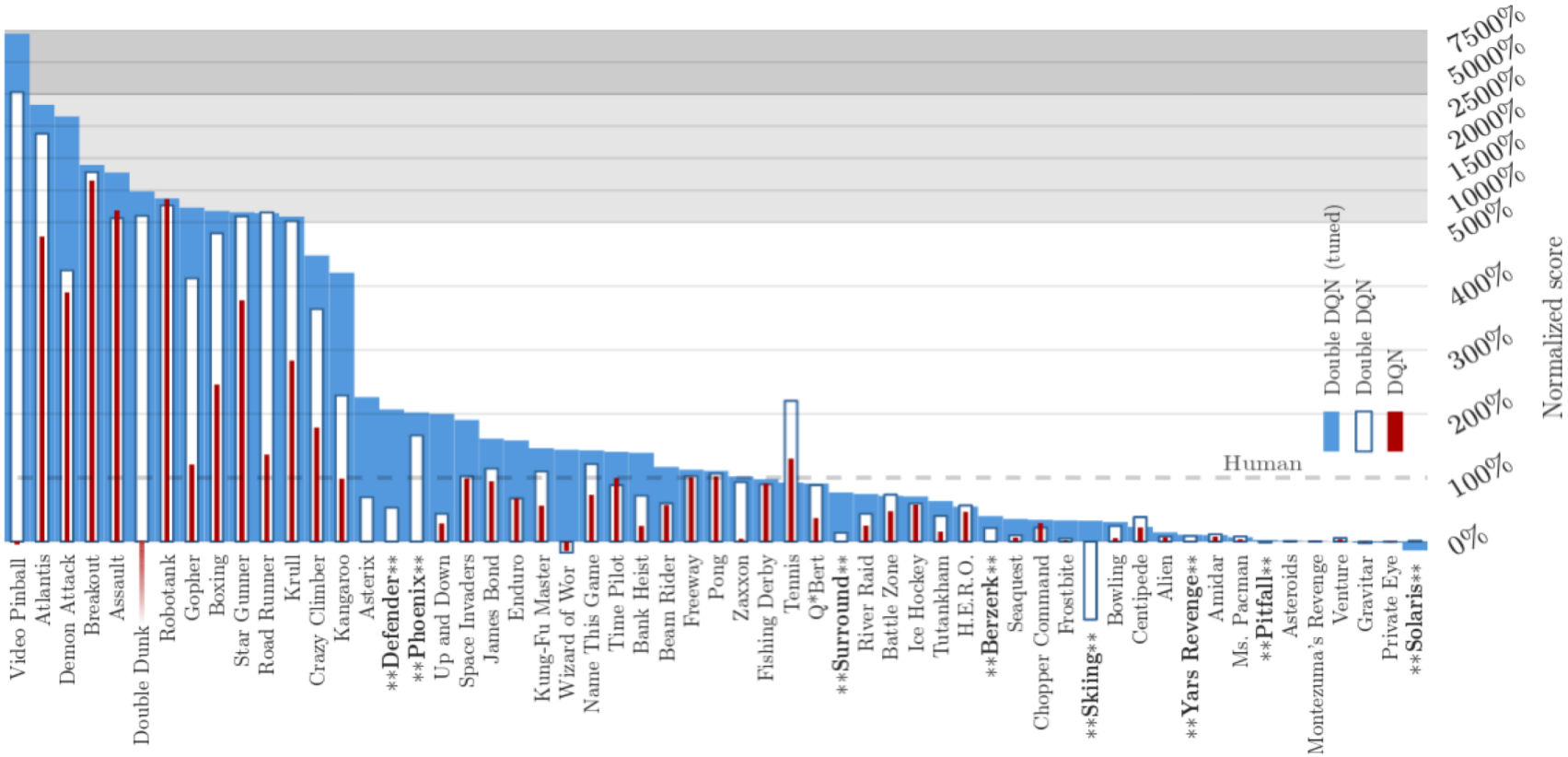
$$y_{\text{target}}(s,a) = r(s) + \gamma \max_{a'} Q_{\text{target}}(s',a')$$

- Double DQN target:

$$y_{\text{target}}(s,a) = r(s) + \gamma\, Q_{\text{target}}\left(s', \operatorname{argmax}_{a'} Q_w(s',a')\right)$$

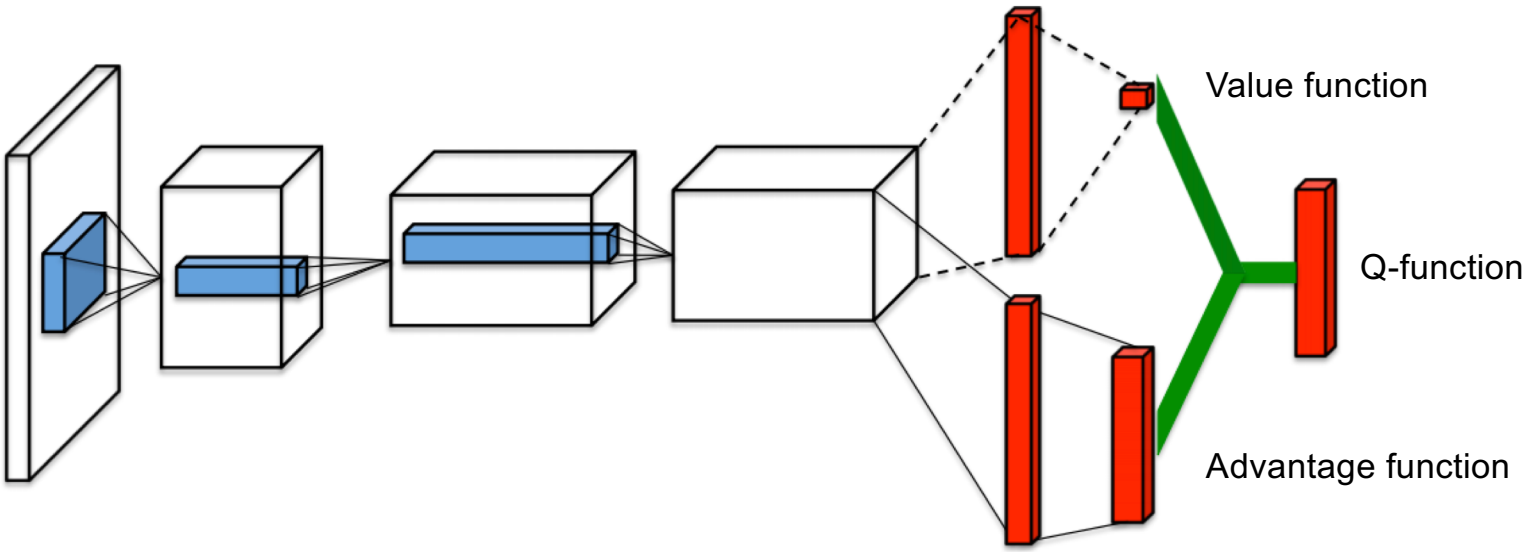
H. van Hasselt, A. Guez, D. Silver, Deep Reinforcement Learning with Double Q-learning, AAAI 2016

Double DQN results



Another extension: Dueling DQN

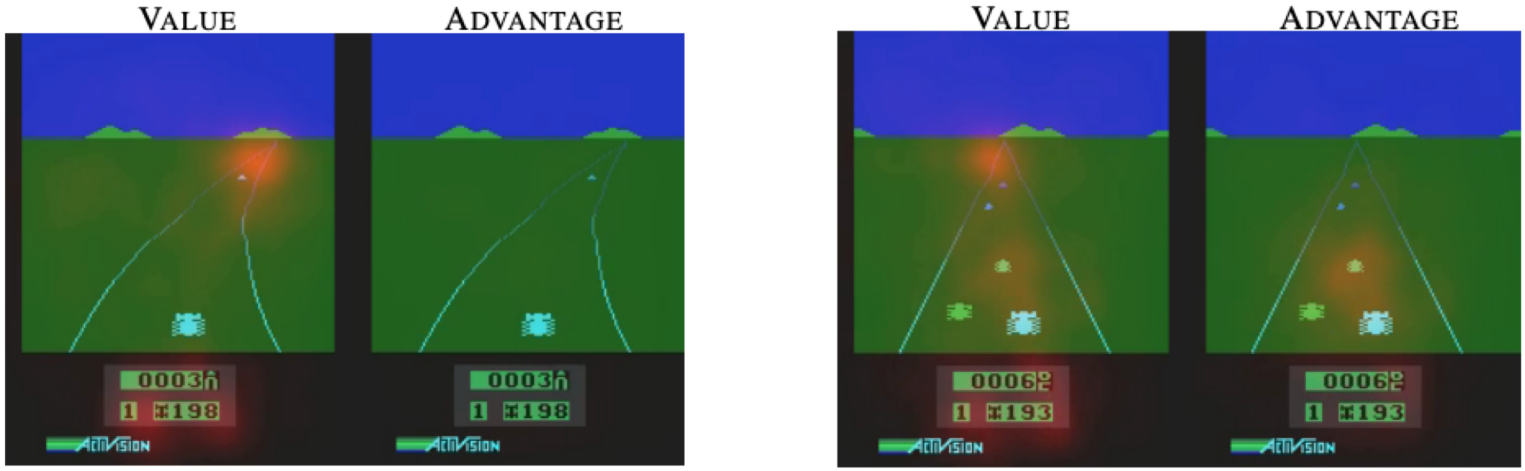
- Decompose estimation of Q-function into *value* and *advantage* functions



Z. Wang et al., Dueling Network Architectures for Deep Reinforcement Learning, ICML 2016

Dueling DQN

- Decompose estimation of Q-function into *value* and *advantage* functions
  - Motivation: in many states, actions don't meaningfully affect the environment, so it is not necessary to know the exact value of each action at each time step



Z. Wang et al., Dueling Network Architectures for Deep Reinforcement Learning, ICML 2016

Dueling DQN

- Decompose estimation of Q-function into *value* and *advantage* functions:

$$Q(s,a) = V(s) + (A(s,a) - \max_{a'} A(s,a'))$$ or

$$Q(s,a) = V(s) + \left(A(s,a) - \frac{1}{|\mathcal{A}|}\sum_{a'} A(s,a')\right)$$

Z. Wang et al., Dueling Network Architectures for Deep Reinforcement Learning, ICML 2016

Dueling DQN: Results

Improvements over prioritized DDQN baseline: