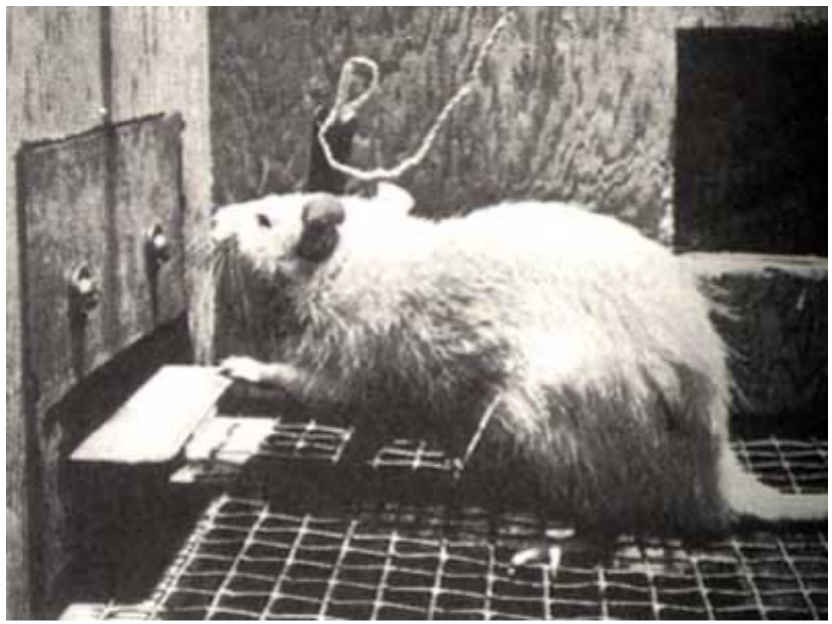
Introduction to deep reinforcement learning

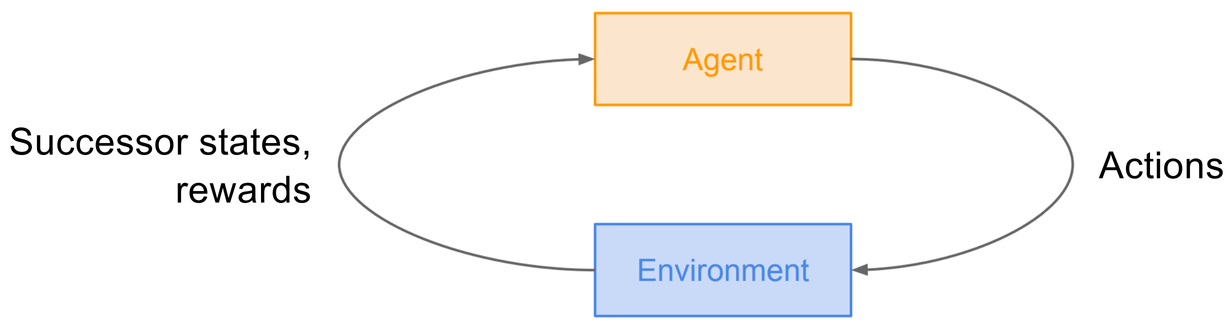


Outline

- Introduction to reinforcement learning
- Markov Decision Process (MDP) formalism
- The Bellman equation
- Q-learning
- Deep Q networks (DQN)
- Extensions
  - Double DQN
  - Dueling DQN

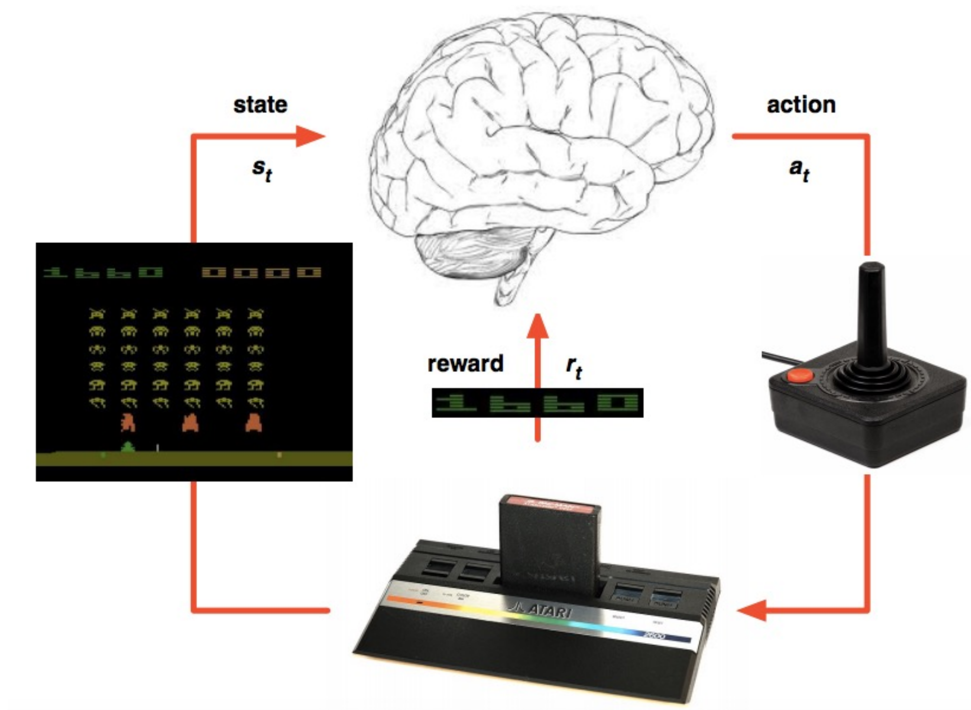
Reinforcement learning (RL)

- Setting: agent that can take *actions* affecting the *state* of the environment and observe occasional *rewards* that depend on the state
- Goal: learn a *policy* (mapping from states to actions) to maximize expected reward over time



Example applications of deep RL

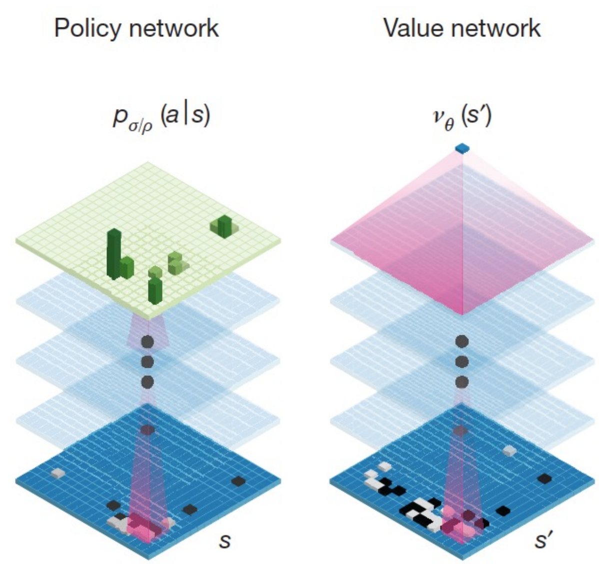
- Playing games



V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller,
Human-level control through deep reinforcement learning, *Nature* 2015

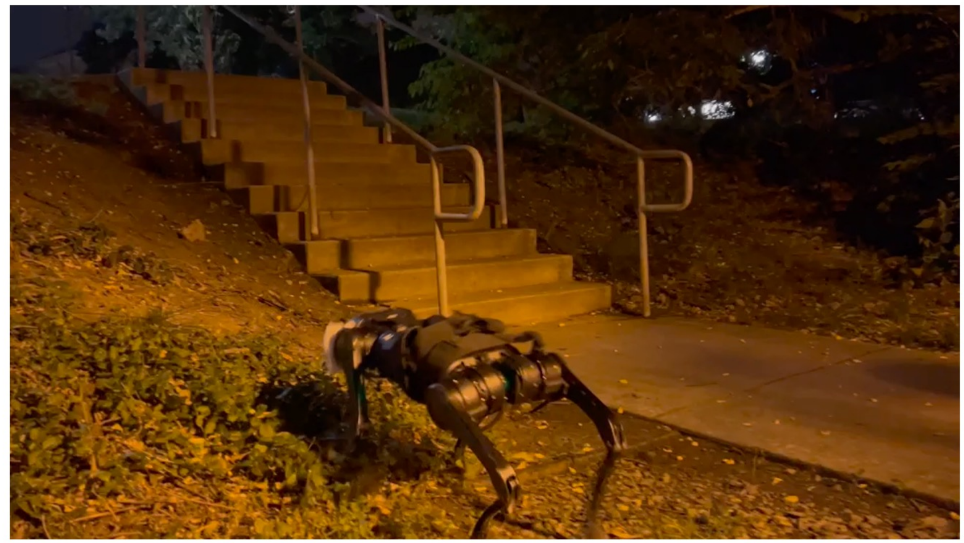
Example applications of deep RL

- Playing games



https://deepmind.com/research/alphago/

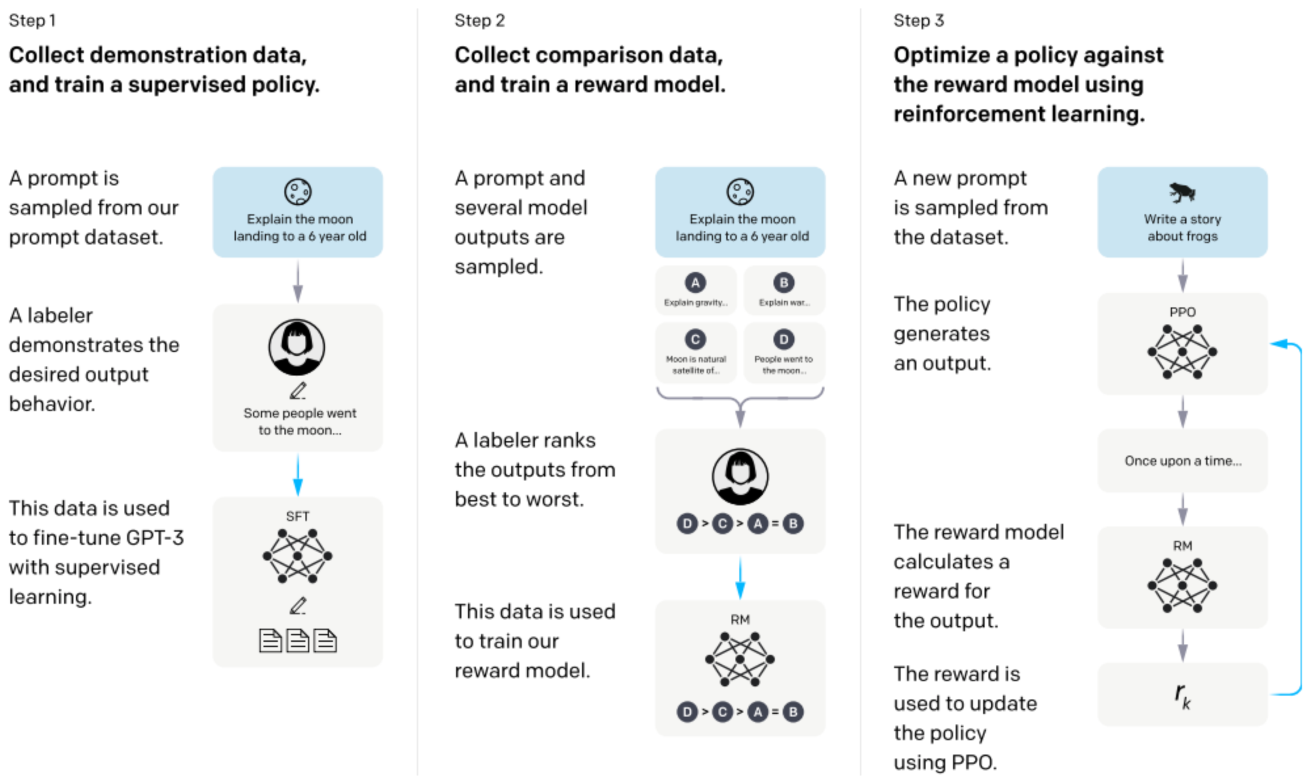
Sensorimotor learning



A. Agarwal, A. Kumar, J. Malik, and D. Pathak. Legged Locomotion in Challenging Terrains using Egocentric Vision. CoRL 2022

Example applications of deep RL

- Improving large language models

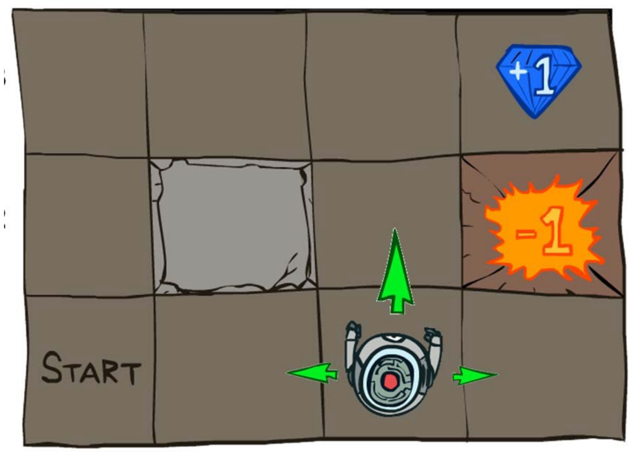


L. Ouyang et al. [Training language models to follow instructions with human feedback](). NeurIPS 2022

Formalism: Markov Decision Processes

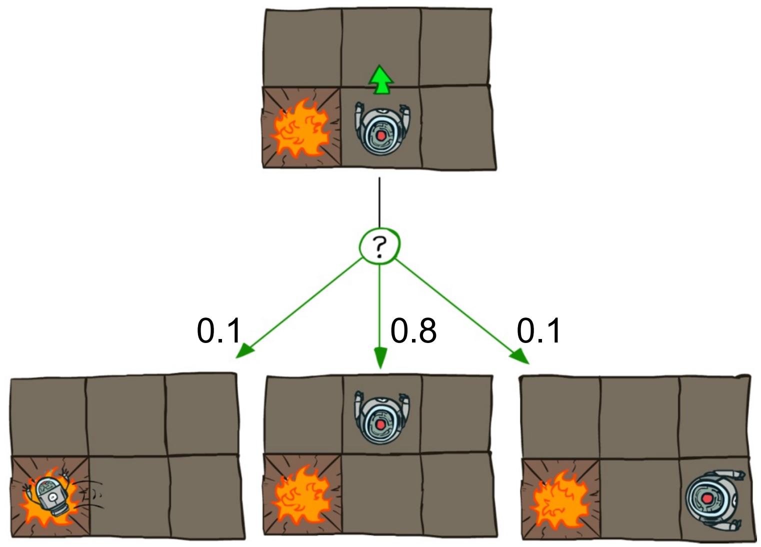
- Components:
  - **States** s, beginning with initial state s_0
  - **Actions** a
  - **Transition model** $P(s' \mid s, a)$
    - *Markov assumption*: the probability of going to s' from s depends only on s and a and not on any other past actions or states
  - **Reward function** $r(s)$
- **Policy** $\pi(s)$: the action that an agent takes in any given state
  - The “solution” to an MDP

Example MDP: Grid world



$r(s) = -0.04$ for every non-terminal state

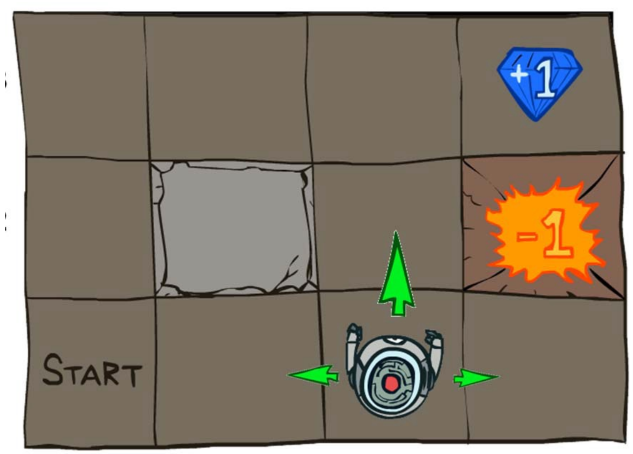
Transition model:

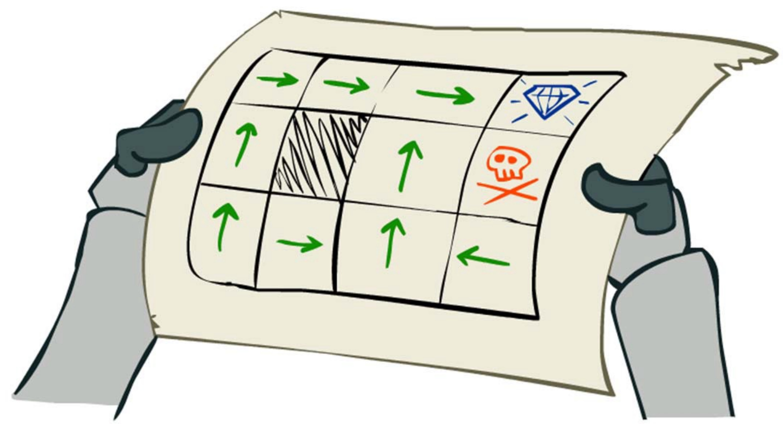


Source: P. Abbeel and D. Klein

Example MDP: Grid world

- Goal: find the best policy

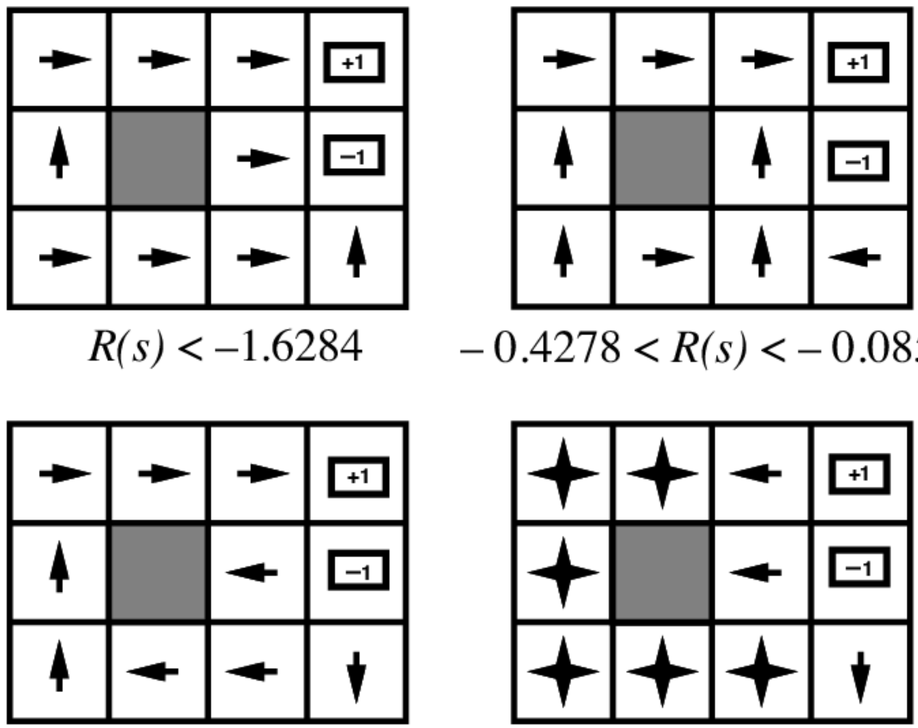




Source: P. Abbeel and D. Klein

Example MDP: Grid world

- Optimal policies for various values of $r(s)$:



$R(s) < -1.6284$

$-0.4278 < R(s) < -0.0850$

$-0.0221 < R(s) < 0$

$R(s) > 0$

Reinforcement learning loop

- From state s, take action a determined by *policy* $\pi(s)$
- Environment selects next state s' based on *transition model* $P(s'|s, a)$
- Observe s' and reward $r(s')$, update policy

- Compare: SGD loop
  - Get input x_i sampled i.i.d. from data distribution
  - Use model with parameters w to predict output y
  - Observe target output y_i and loss $l(w, x_i, y_i)$
  - Update w to reduce loss: $w \leftarrow w - \eta \nabla l(w, x_i, y_i)$

RL vs. supervised learning

- **Reinforcement learning**
  - Agent's actions affect the environment and help to determine next observation
  - Rewards may be sparse
  - Rewards are *not differentiable* w.r.t. model parameters

- **Supervised learning**
  - Next input does not depend on previous inputs or agent's predictions
  - There is a supervision signal at every step
  - Loss is differentiable w.r.t. model parameters

Solving MDPs

- Components:
  - **States** s, beginning with initial state s_0
  - **Actions** a
  - **Transition model** $P(s' \mid s, a)$
    - *Markov assumption*: the probability of going to s' from s depends only on s and a and not on any other past actions or states
  - **Reward function** $r(s)$
- **Policy** $\pi(s)$: the action that an agent takes in any given state
  - The "solution" to an MDP
  - But how to find this solution?

Cumulative rewards of state sequences

- Suppose that following policy π starting in state s_0 leads to a *sequence* or *trajectory* $\tau = (s_0, s_1, s_2, \dots)$
- How do we define the cumulative reward of this trajectory?
- **Problem:** the sum of rewards of individual states grows is not normalized w.r.t. sequence length and can even be infinite
- **Solution:** define cumulative reward as sum of rewards *discounted* by a factor γ, $0 < \gamma \leq 1$

Image source: P. Abbeel and D. Klein

Discounting

- *Discounted cumulative reward* of trajectory $\tau = (s_0, s_1, s_2, s_3, \dots)$:

$$r(s_0, s_1, s_2, s_3, \dots) = r(s_0) + \gamma\, r(s_1) + \gamma^2 r(s_2) + \gamma^3 r(s_3) + \cdots$$
$$= \sum_{t \geq 0} \gamma^t\, r(s_t)$$

- Sum is bounded by $\frac{r_{\max}}{1-\gamma}$ (assuming $0 < \gamma \leq 1$)
- Helps algorithms converge
- Notice:

$$r(s_0, s_1, s_2, s_3, \dots) = r(s_0) + \gamma\, r(s_1, s_2, s_3, \dots)$$

Cumulative reward of trajectory starting at s_0	Reward at s_0	Discounted reward of trajectory starting at s_1

Value function

- The *value function* $V^{\pi}(s)$ of a state s w.r.t. policy π is the expected cumulative reward of following that policy starting in s:

$$V^{\pi}(s) = \mathbb{E}_{\tau}[r(\tau)|\ s_0 = s, \pi]$$

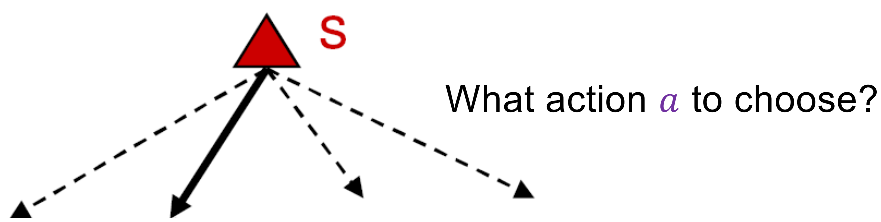
where τ is a trajectory with starting state s, actions given by π, and successor states drawn according to transition model: $s_{t+1} \sim P(\cdot\,|s_{t,}a_t)$

- The *optimal value* of a state is the value achievable by following the best possible policy:

$$V^{*}(s) = \max_{\pi} V^{\pi}(s)$$

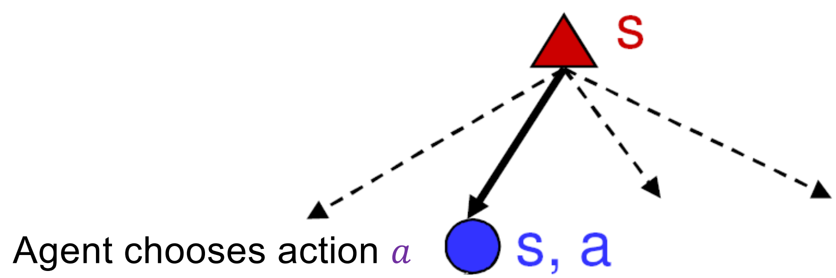
The optimal policy

- How do we express the optimal policy in terms of optimal state values?



The optimal policy

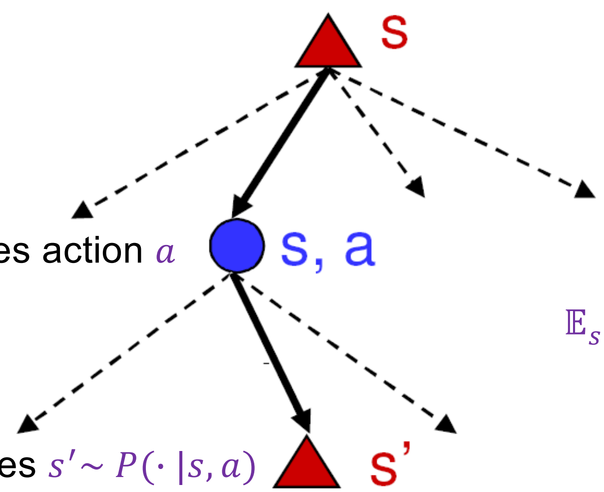
- How do we express the optimal policy in terms of optimal state values?



The optimal policy

- How do we express the optimal policy in terms of optimal state values?
  - Take the action that maximizes the expected future cumulative value:

$$\pi^*(s) = \arg\max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$



Agent chooses action a

Environment chooses $s' \sim P(\cdot\,|s,a)$

Expected reward: $V^*(s')$

Expected value for action a:

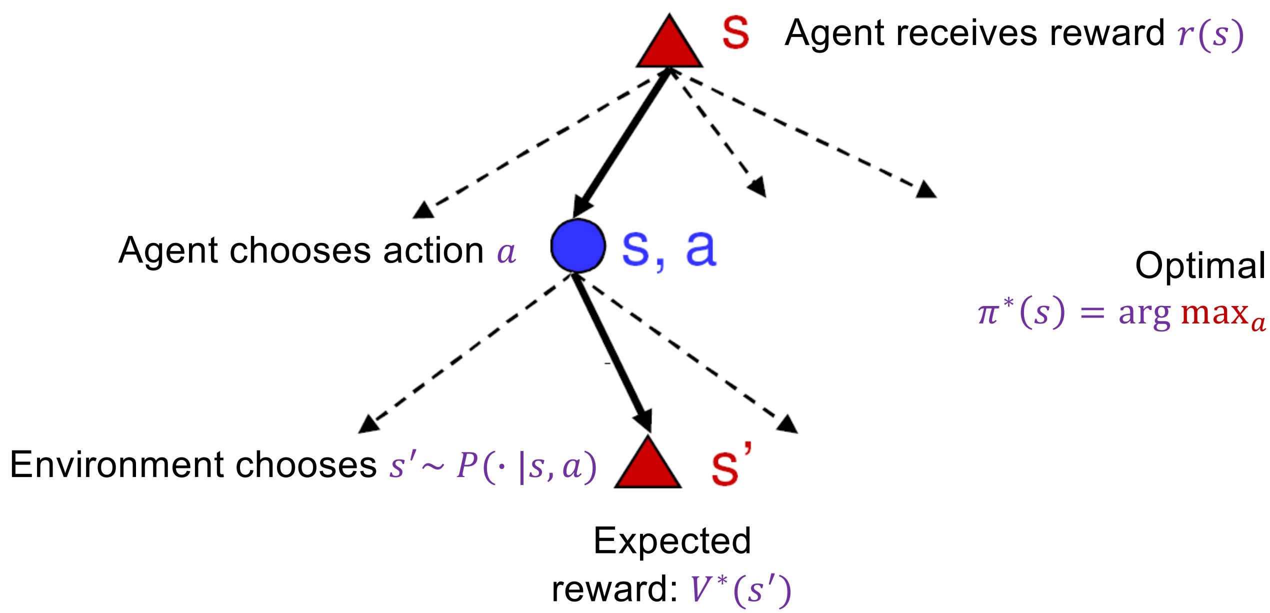
$$\mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s') = \sum_{s'} P(s'|s,a) V^*(s')$$

The Bellman equation

$$V^*(s) \;=\; r(s) \;+\; \gamma \max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$

Reward in current state

Expected future cumulative value assuming agent follows the optimal policy



Optimal policy:

$\pi^*(s) = \arg\max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$

The Bellman equation

$$V^*(s) \quad = \quad r(s) \quad + \quad \gamma \underbrace{\max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')}$$

Reward in current state

Expected future cumulative value assuming agent follows the optimal policy

- It's a recursive relationship between optimal values of successive states!

Q-learning

- To choose actions using value functions, we need to know the transition model:

$$\pi^*(s) = \arg\max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$
$$= \arg\max_a \sum_{s'} P(s'|s,a) V^*(s')$$

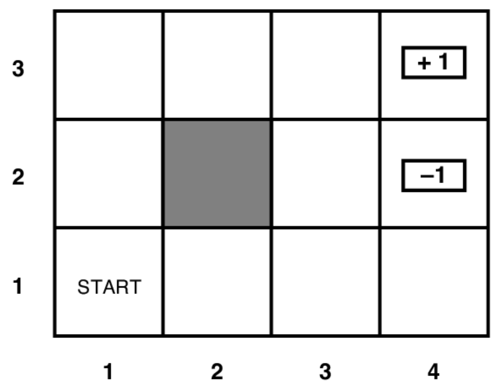
- It is more convenient to define the value of a *state-action pair*:

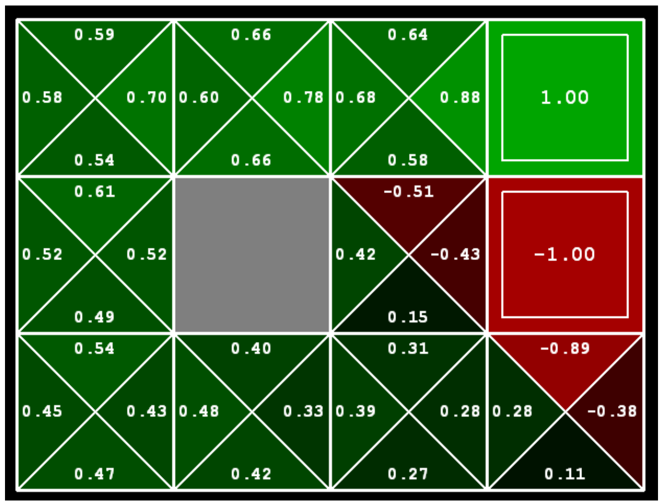
$$Q^\pi(s,a) = \mathbb{E}_\tau[r(\tau)|s_0 = s,\, a_0 = a, \pi]$$

- Then the optimal policy is given by

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

Q-value function: Example





Bellman equation for Q-values

- Relationship between regular values and Q-values:

$$V^*(s) = \max_a Q^*(s,a)$$

- Regular Bellman equation:

$$V^*(s) = r(s) + \gamma \max_a \mathbb{E}_{s' \sim P(\cdot|s,a)} V^*(s')$$

- Bellman equation for Q-values:

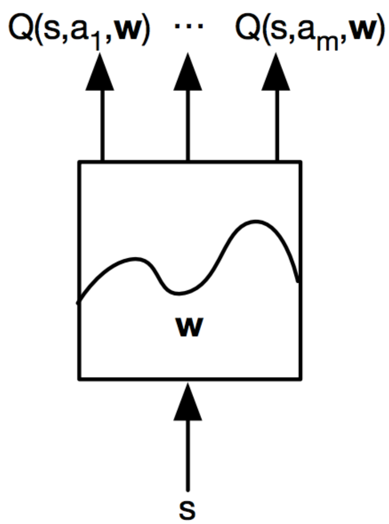
$$Q^*(s,a) = r(s) + \gamma\, \mathbb{E}_{s' \sim P(\cdot|s,a)} \max_{a'} Q^*(s',a')$$
$$= \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[r(s) + \gamma \max_{a'} Q^*(s',a') \right]$$

Deep Q-learning

$$Q^*(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[r(s) + \gamma \max_{a'} Q^*(s',a') | s,a \right]$$

- **Idea 1**: estimate Q-values using a neural network:

$$Q^*(s,a) \approx Q_w(s,a)$$



V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller, Human-level control through deep reinforcement learning, *Nature* 2015

Deep Q-learning

$$Q^*(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[r(s) + \gamma \max_{a'} Q^*(s',a') | s,a \right]$$

- **Idea 1**: estimate Q-values using a neural network:

$$Q^*(s,a) \approx Q_w(s,a)$$

- **Idea 2**: instead of taking the expectation over successor states, use a single *transition* (s,a,s') to update estimate of $Q_w(s,a)$ to better agree with the *target*

$$r(s) + \gamma \max_{a'} Q_w(s',a')$$

- **Idea 3**: to avoid training instability, compute target values using a different snapshot of the network, Q_{target}

Deep Q-learning

- Given transition (s, a, s'), compute target:

$$y_{\text{target}}(s, a, s') = r(s) + \gamma \max_{a'} Q_{\text{target}}(s', a')$$

- Loss function:

$$L(w) = \mathbb{E}_{s,a,s'} \left[\left(y_{\text{target}}(s, a, s') - Q_w(s, a) \right)^2 \right]$$

- Gradient update:

$$\nabla_w L(w) = \mathbb{E}_{s,a,s'} \left[\left(y_{\text{target}}(s, a, s') - Q_w(s, a) \right) \nabla_w Q_w(s, a) \right]$$

$$= \mathbb{E}_{s,a,s'} \left[\left(r(s) + \gamma \max_{a'} Q_{\text{target}}(s', a') - Q_w(s, a) \right) \nabla_w Q_w(s, a) \right]$$

- Sample transitions (s, a, s') by choosing actions from a *behavior distribution* and using an *experience replay buffer*

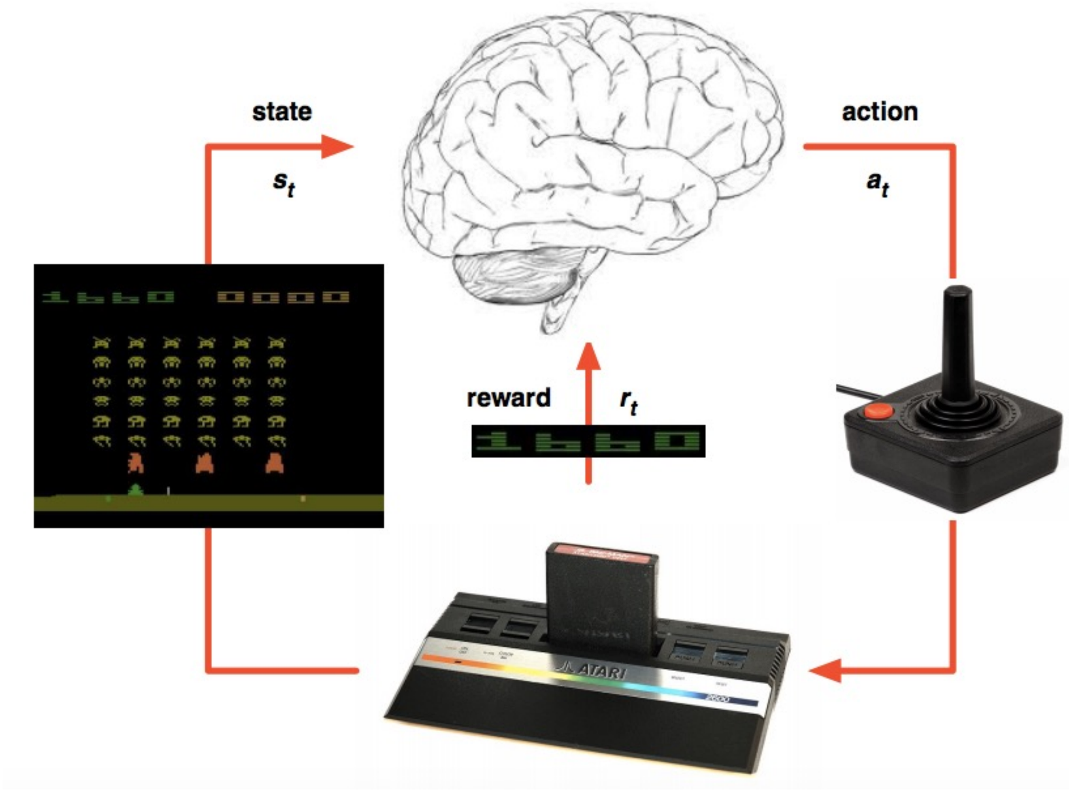
Deep Q-learning: Summary

- At each time step:
  - Take action a_t according to *epsilon-greedy policy*
  - Store experience $(s_t, a_t, r_{t+1}, s_{t+1})$ in *replay memory buffer*

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\ldots$
$s_t, a_t, r_{t+1}, s_{t+1}$

  - Randomly sample *mini-batch* of experiences from the buffer
  - Perform gradient descent step on loss:
    $$L(w) = \mathbb{E}_{s,a,s'}\left[\left(r(s) + \gamma \max_{a'} Q_{\text{target}}(s', a') - Q_w(s,a)\right)^2\right]$$
  - Update target network every C steps

Deep Q-learning in Atari

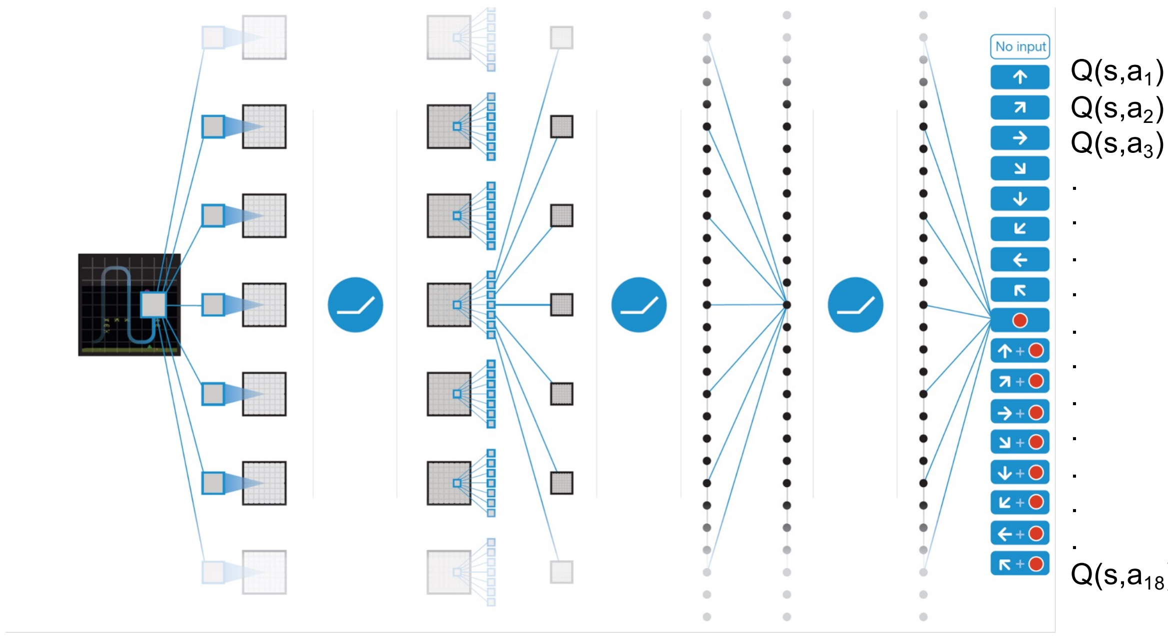


V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller,
[Human-level control through deep reinforcement learning](), *Nature* 2015

Deep Q-learning in Atari

- End-to-end learning of $Q(s,a)$ from pixels s
- Output is $Q(s,a)$ for 18 joystick/button configurations
- Reward is change in score for that step

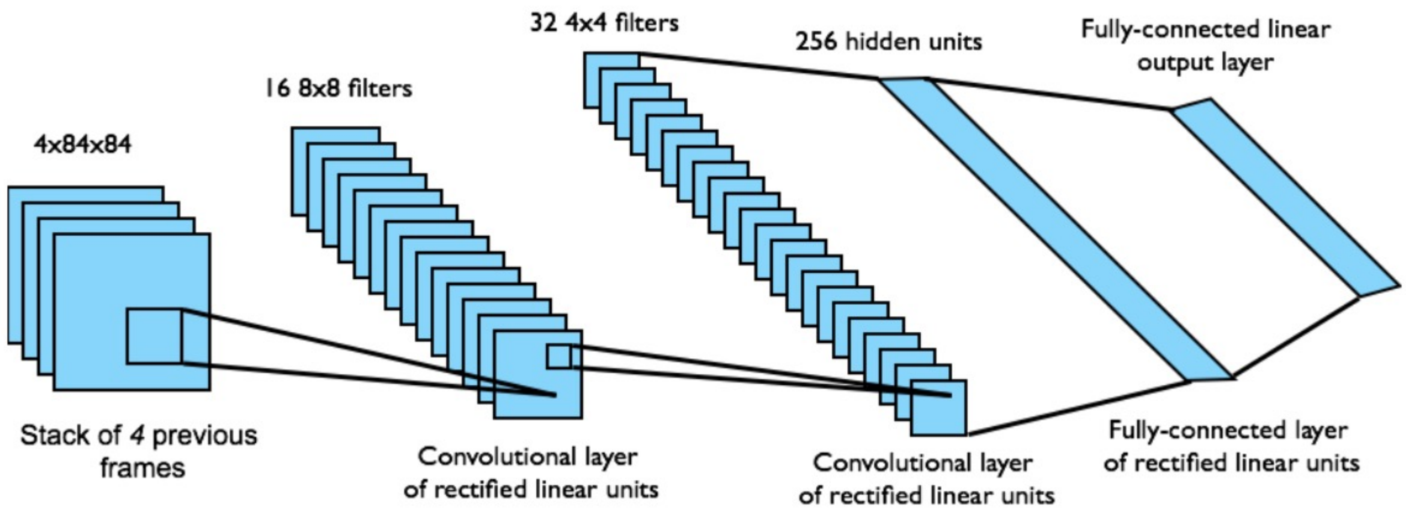
Deep Q-Network (DQN)



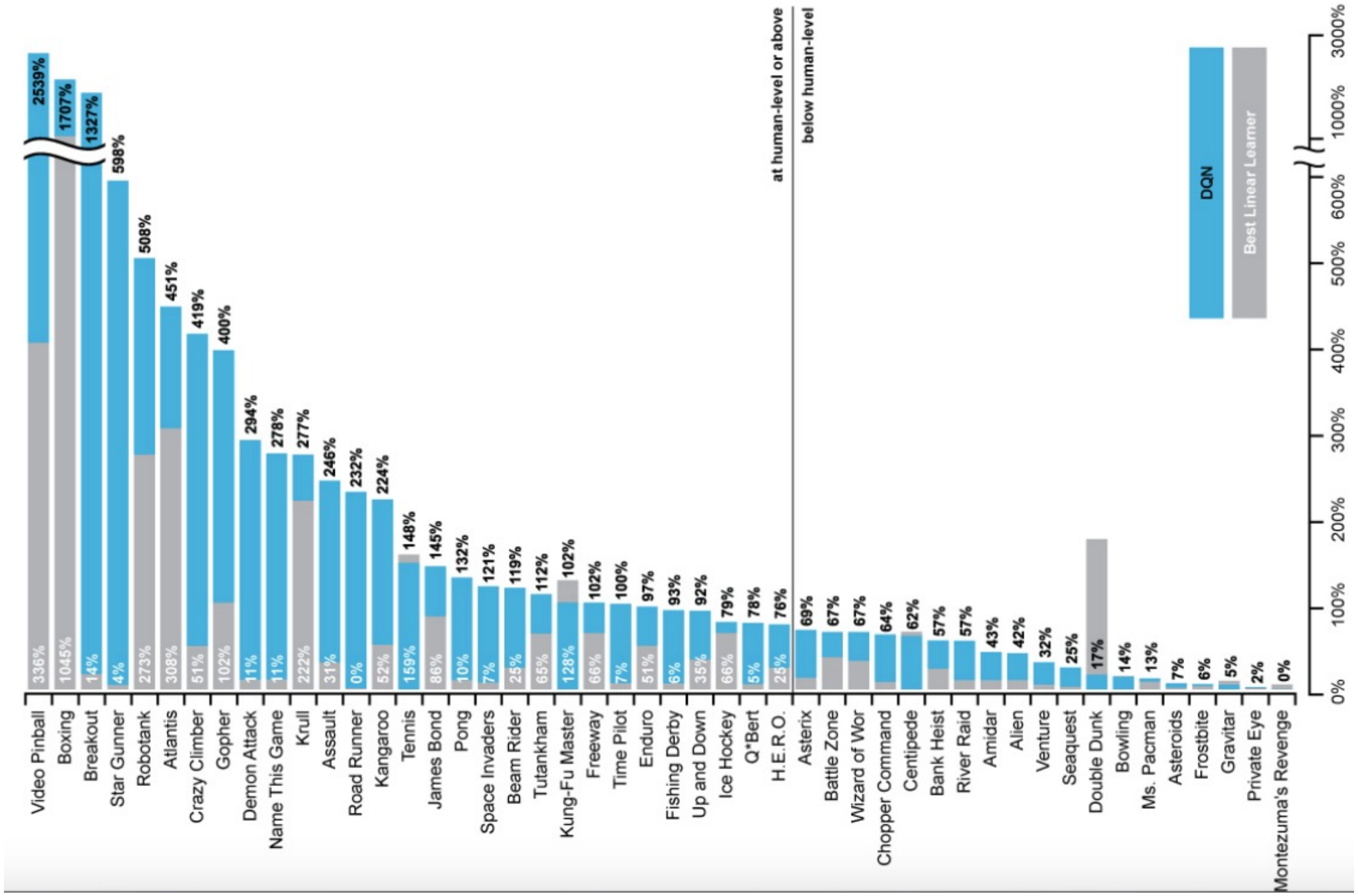
Deep Q-learning in Atari

- Input state is stack of raw pixels (grayscale) from last 4 frames
- Network architecture and hyperparameters fixed for all games

Deep Q-Network (DQN)



Deep Q-learning in Atari



Breakout demo

https://www.youtube.com/watch?v=TmPfTpjtdgg

Outline

- Introduction to reinforcement learning
- Markov Decision Process (MDP) formalism and classical Bellman equation
- Q-learning
- Deep Q networks
- Extensions
  - Double DQN
  - Dueling DQN

Extension: Double Q-learning

- Max operator in standard Q-learning is used both to select and evaluate an action, leading to systematic over-estimation of Q-values
- Modification: *select* action using the online (current) network, but *evaluate* Q-value using the target network
- Regular DQN target:

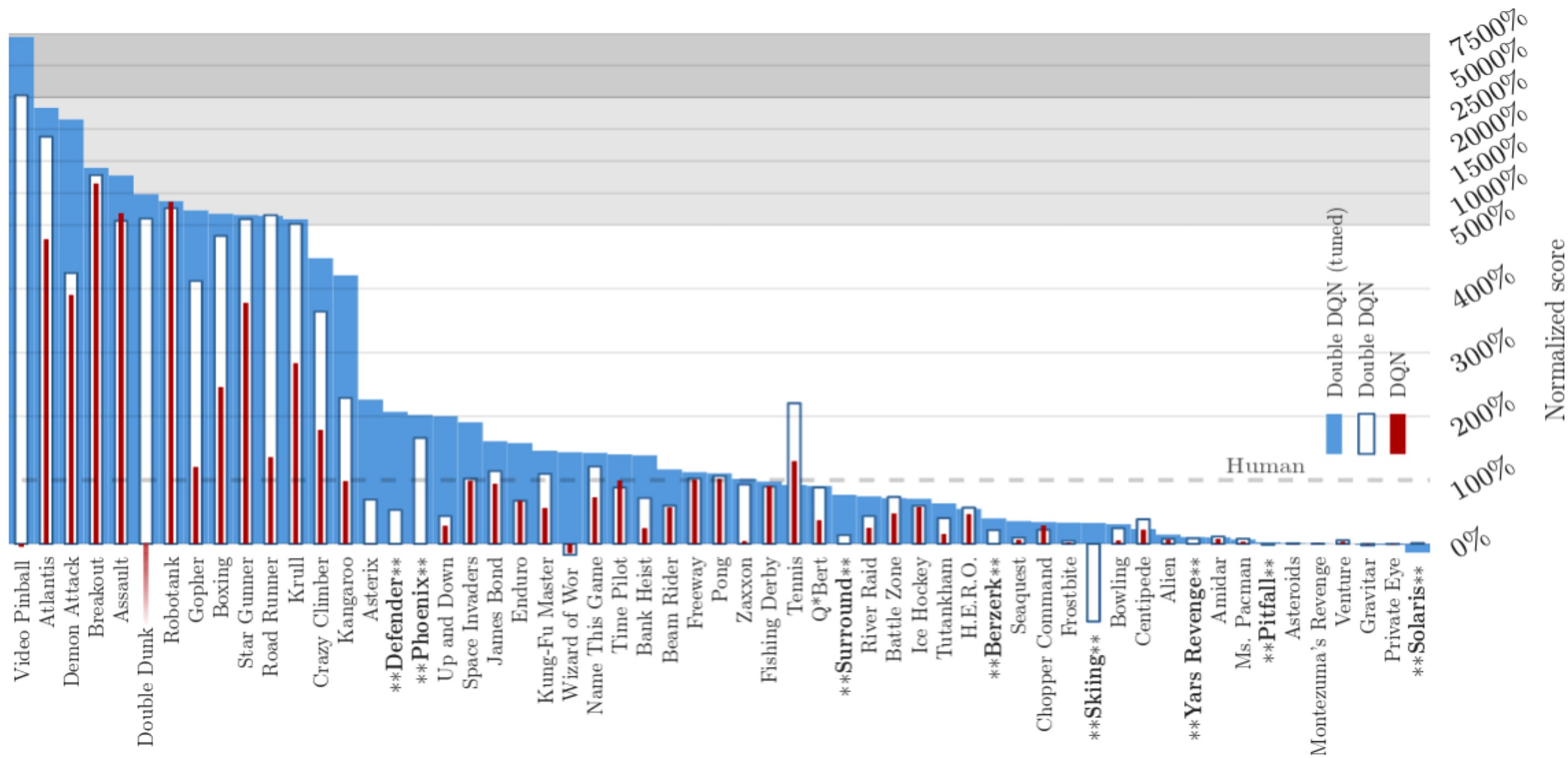
$$y_{\text{target}}(s, a) = r(s) + \gamma \max_{a'} Q_{\text{target}}(s', a')$$

- Double DQN target:

$$y_{\text{target}}(s, a) = r(s) + \gamma\, Q_{\text{target}}\left(s', \operatorname{argmax}_{a'} Q_w(s', a')\right)$$

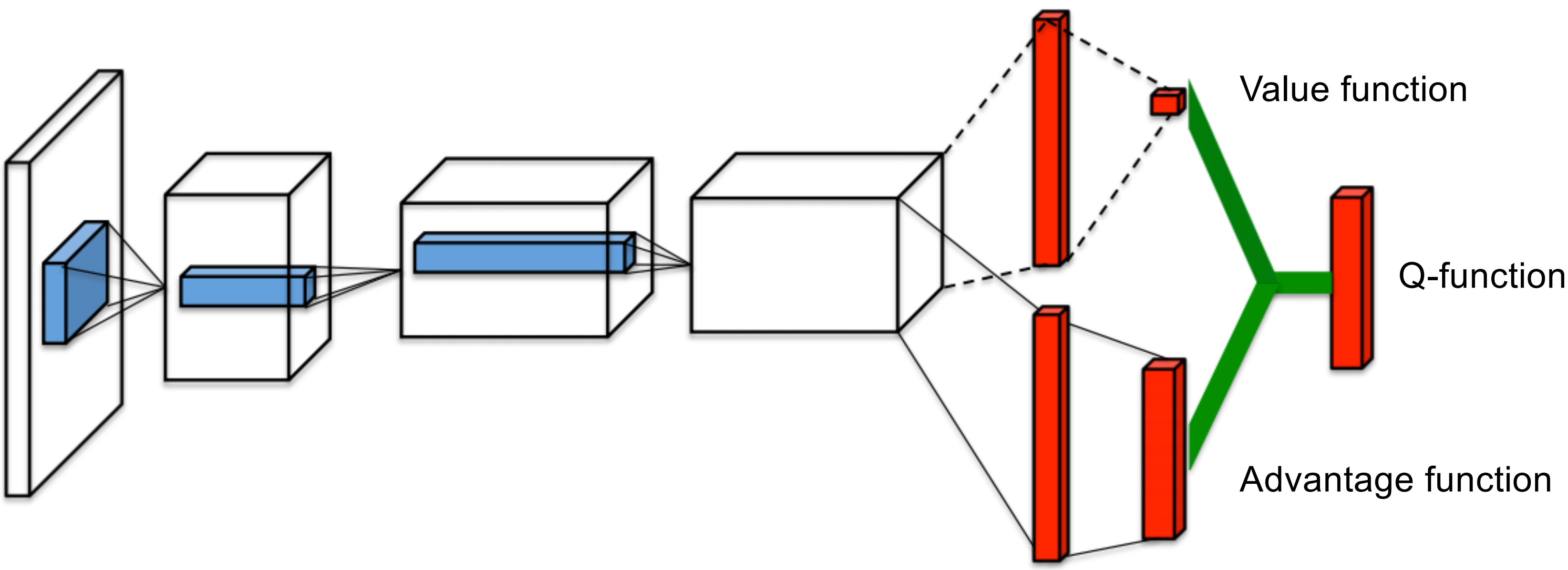
H. van Hasselt, A. Guez, D. Silver, [Deep Reinforcement Learning with Double Q-learning](), AAAI 2016

Double DQN results



Another extension: Dueling DQN

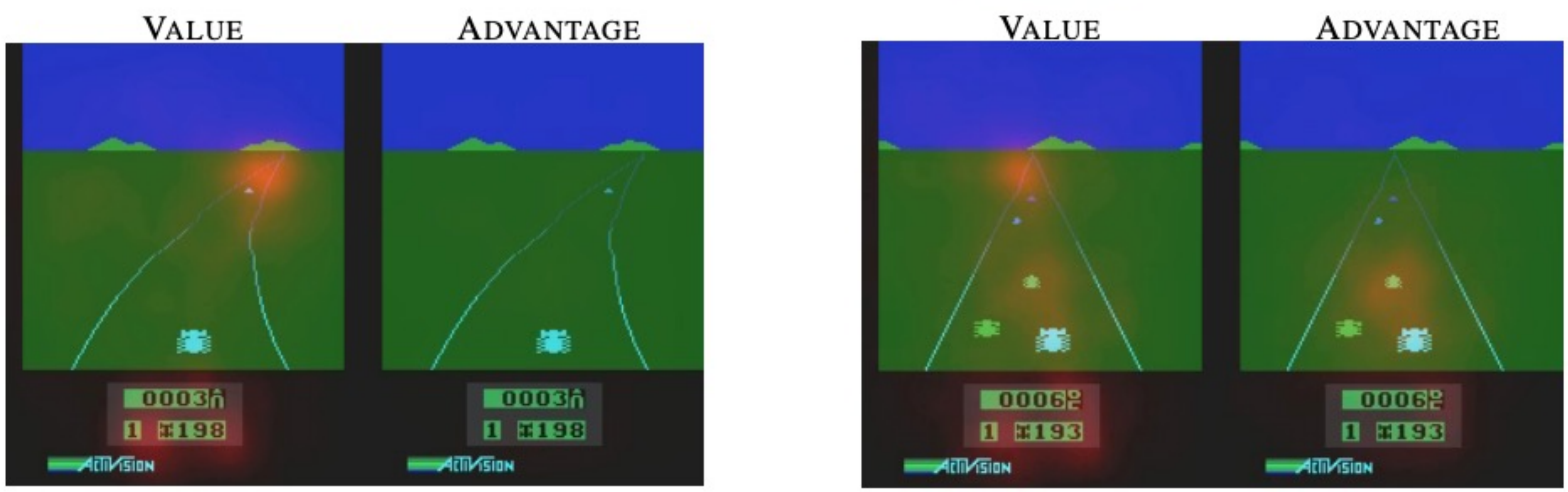
- Decompose estimation of Q-function into *value* and *advantage* functions



Z. Wang et al., Dueling Network Architectures for Deep Reinforcement Learning, ICML 2016

Dueling DQN

- Decompose estimation of Q-function into *value* and *advantage* functions
  - Motivation: in many states, actions don't meaningfully affect the environment, so it is not necessary to know the exact value of each action at each time step



Z. Wang et al., [Dueling Network Architectures for Deep Reinforcement Learning](), ICML 2016

Dueling DQN

- Decompose estimation of Q-function into *value* and *advantage* functions:

$$Q(s,a) = V(s) + \left(A(s,a) - \max_{a'} A(s,a')\right) \text{ or}$$

$$Q(s,a) = V(s) + \left(A(s,a) - \frac{1}{|\mathcal{A}|}\sum_{a'} A(s,a')\right)$$

Z. Wang et al., [Dueling Network Architectures for Deep Reinforcement Learning](), ICML 2016

Dueling DQN: Results

Improvements over prioritized DDQN baseline: